

## 2024 TKGS PRELIM MATH P1 SOLUTION

Qn		Solution	Content/Success Criteria	
1	(a)	8700	I can perform calculations with a calculator.	
			I can round off values to the nearest hundred.	
	(b)	1050	Content	L
			Complexity	L
			Context	L
			Response Strategy	L
			Assessment Objective	AO1

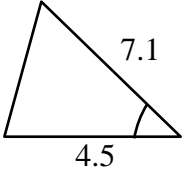
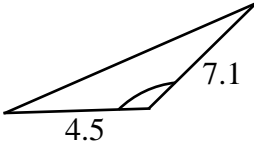
Qn		Solution	Content/Success Criteria	
2	(a)	$5 - 2(6x - 1)$ $= 5 - 12x + 2$ $= 7 - 12x$	I can expand and simplify algebraic expressions.	
			I can apply simplify expression using laws of indices.	
			Content	L
			Complexity	L
			Context	L
			Response Strategy	L
	(b)	$\left(\frac{27b^9}{a^6}\right)^{-\frac{1}{3}} = \left(\frac{a^6}{27b^9}\right)^{\frac{1}{3}}$ $= \frac{a^2}{3b^3}$	Assessment Objective	AO1

Qn	Solution	Content/Success Criteria	
3	Let amount of savings Albert and Chris have initially be $5x$ and $3x$ respectively.  $\frac{5x-30}{3x-30} = \frac{2}{1}$ $5x-30 = 2(3x-30)$ $5x-30 = 6x-60$ $x = 30$ $3x = 90$ The amount of savings Chris has at the start = \$90	Content	L
		Complexity	L
		Context	L
		Response Strategy	M
		Assessment Objective	AO2

Qn	Solution	Content/Success Criteria	
4	(a) $1400 = 2^3 \times 5^2 \times 7$  (b) $1400 = 2^3 \times 5^2 \times 7$ $q = 2^2 \times 3 \times 7 = 84$ $HCF = 2^2 \times 7$  $\therefore q = 84$	I can express a number in its prime factors.  I can use the prime factors from HCF to find the original number.	
		Content	L
		Complexity	H
		Context	L
		Response Strategy	M
		Assessment Objective	AO1/ AO2

Qn	Solution	Content/Success Criteria	
<b>5</b>	$\frac{1}{2^{a-1}} = 2^3 + 2^3 + 2^3 + 2^3$ $\frac{1}{2^{a-1}} = 32$ $2^{1-a} = 2^5$ <p>Comparing indices,</p> $1 - a = 5$ $a = -4$	I can solve equation using laws of indices. $a^m \times a^n = a^{m+n}$ $a^{-n} = \frac{1}{a^n}$	
		Content	L
		Complexity	H
		Context	L
		Response Strategy	H
		<b>Assessment Objective</b>	AO2
<b>OR</b>	$\frac{1}{2^{a-1}} = 2^3 + 2^3 + 2^3 + 2^3$ $\frac{1}{2^{a-1}} = 2^3(1+1+1+1)$ $\frac{1}{2^{a-1}} = 2^3(2^2)$ $\frac{1}{2^{a-1}} = 2^5$ $2^{-(a-1)} = 2^5$ $-a + 1 = 5$ $a = -4$		

Qn	Solution	Content/Success Criteria	
<b>6</b>	<p><b>(a)</b></p> $2(5m + 3n)^2$ $= 2(25m^2 + 30mn + 9n^2)$ $= 50m^2 + 60mn + 18n^2$ <p><b>(b)</b></p> $24(mn)^2 - 21mn^3$ $= 24m^2n^2 - 21mn^3$ $= 3mn^2(8m - 7n)$	I can use identity $(a+b)^2 = a^2 + 2ab + b^2$ to expand algebraic expressions.	
		I can factorise algebraic expressions by taking out common factor.	
		Content	L
		Complexity	M
		Context	L
		Response Strategy	M
		<b>Assessment Objective</b>	AO1

Qn	Solution	Content/Success Criteria	
7	$\frac{1}{2}(4.5)(7.1)\sin \angle XYZ = 12.6$ $15.975 \sin \angle XYZ = 12.6$ $\sin \angle XYZ = \frac{12.6}{15.975}$ $\sin \angle XYZ = \frac{56}{71}$ <div style="display: flex; justify-content: space-around; align-items: center;">   </div> $\angle XYZ = \sin^{-1}\left(\frac{12.6}{15.975}\right) \quad \text{or} \quad \angle XYZ = \pi - \sin^{-1}\left(\frac{12.6}{15.975}\right)$ $\angle XYZ = 0.909 \text{ radian} \quad \quad \quad \angle XYZ = 2.23 \text{ radian}$	I can use $\frac{1}{2}ab \sin C$ for area of triangle to find sine of acute and obtuse angles in radians.	
		Content	L
		Complexity	M
		Context	L
		Response Strategy	L
		Assessment Objective	AO2

Qn	Solution	Content/Success Criteria	
8	$P = \frac{k}{Q^3}$ , where $k$ is a non-zero constant $Q$ is reduced by 20%, substitute original $Q$ with $0.8Q$ . $P_{\text{new}} = \frac{k}{(0.8Q)^3}$ $= \frac{k}{0.512Q^3}$ New percentage of $P = \frac{1}{0.512} \times 100\%$ $= 195 \frac{5}{16} \%$ Percentage change in $P = 95 \frac{5}{16} \%$ or 95.3125%	I can use inverse proportion to find percentage change.	
		Content	L
		Complexity	M
		Context	L
		Response Strategy	L
OR	$Q_{\text{new}} = 80\% \text{ of } Q$ $= \frac{4}{5}Q$ $P_{\text{new}} \left( \frac{4}{5}Q \right)^3 = PQ^3$ $P_{\text{new}} = \frac{125}{64}P$ Percentage change in $P = \frac{\frac{125}{64}P - P}{P} \times 100\%$ $= \frac{61}{64} \times 100\%$ $= 95 \frac{5}{16} \%$ or 95.3125%	Assessment Objective	AO2

Qn		Solution	Content/Success Criteria	
9	(a)	$p = \frac{\frac{1}{5} + (-6)^2}{\frac{1}{5} - 5}$ $= -\frac{181}{24} \text{ or } -7\frac{13}{24} \text{ or } -7.54$	I can evaluate algebraic expressions by substitution.	
	(b)	$p = \frac{q + r^2}{q - 5}$ $p(q - 5) = q + r^2$ $pq - 5p = q + r^2$ $pq - q = r^2 + 5p$ $q(p - 1) = r^2 + 5p$ $q = \frac{r^2 + 5p}{p - 1}$	I can change the subject of the formula.	
			Content	L
			Complexity	L
			Context	L
			Response Strategy	L
			Assessment Objective	AO1

Qn		Solution	Content/Success Criteria	
10	(a)	$\sin 30^\circ = \frac{BD}{32}$ <div style="border: 1px solid black; padding: 2px; display: inline-block;"> <math>DB = 32 \sin 30^\circ</math> </div> <b>Essential step</b> $= 16 \text{ cm}$	I can use TOA CAH SOH to find unknown sides in right-angled triangle.	
			Content	L
			Complexity	L
			Context	L
			Response Strategy	L
			Assessment Objective	AO1
	(b)	$AD^2 + BD^2 = 12^2 + 16^2$ $= 400$ $AB^2 = 20^2$ $= 400$ <p>Since <math>AD^2 + BD^2 = AB^2</math>, angle <math>ADB = 90^\circ</math> by the converse of Pythagoras' Theorem.</p> <p>Since angle <math>ADB = 90^\circ</math>, by the property <b>right angle in semicircle</b>, it is possible to draw diameter <math>AB</math> such that point <math>D</math> lies on the circumference of the circle.</p>	I can use converse of Pythagoras Theorem to prove right angles.	
			I can use right-angle in semicircle to determine that the points in triangle lie on the circumference of circle.	
			Content	H
			Complexity	H
			Context	L
			Response Strategy	H
			Assessment Objective	AO3

Qn	Solution		Content/Success Criteria	
11	$\frac{3}{(1-2h)^2} + \frac{8}{2h-1}$ $= \frac{3}{(1-2h)^2} - \frac{8}{1-2h}$ $= \frac{3-8(1-2h)}{(1-2h)^2}$ $= \frac{3-8+16h}{(1-2h)^2}$ $= \frac{-5+16h}{(1-2h)^2}$	<p>Alternatively,</p> $\frac{3}{(1-2h)^2} + \frac{8}{2h-1}$ $= \frac{3}{(2h-1)^2} + \frac{8}{2h-1}$ $= \frac{3+8(2h-1)}{(1-2h)^2}$ $= \frac{3+16h-8}{(1-2h)^2}$ $= \frac{16h-5}{(1-2h)^2}$	I can add two algebraic fractions with quadratic denominators.	
			Content	L
			Complexity	M
			Context	L
			Response Strategy	M
			Assessment Objective	AO1

Qn	Solution		Content/Success Criteria	
12	(a)	$2k - 5$	I can form and solve linear inequalities in one variable.	
			Content	L
			Complexity	L
			Context	L
			Response Strategy	L
			Assessment Objective	AO1
12	(b)	$2k - 5 + 2k - 3 > 16$ $4k - 8 > 16$ $4k > 24$ $k > 6$  Smallest possible value of the larger odd number = $2(7) - 3 = 11$		
			Content	L
			Complexity	L
			Context	L
			Response Strategy	L
			Assessment Objective	AO1



Qn	Solution	Content/Success Criteria	
<b>13</b>	$OA = OB = OD$ (radius of circle) Angle $BAO = x^\circ$ (base $\angle$ s of isos $\Delta$ ) Angle $AOB = 180^\circ - 2x^\circ$ ( $\angle$ sum of isosceles $\Delta$ ) Angle $DAO = y^\circ$ (base $\angle$ s of isos $\Delta$ ) Angle $AOD = 180^\circ - 2y^\circ$ ( $\angle$ sum of isosceles $\Delta$ )  $\text{Angle } BCD = \frac{1}{2}[(180^\circ - 2x^\circ) + (180^\circ - 2y^\circ)]$ $(\angle \text{ at centre} = 2 \angle \text{ at circumference})$ $= \frac{1}{2}(360^\circ - 2x^\circ - 2y^\circ)$ $= 180^\circ - x^\circ - y^\circ$	I can use angle properties of circles to find an unknown angle.	
		Content	L
		Complexity	L
		Context	L
		Response Strategy	M
		<b>Assessment Objective</b>	AO2
<b>OR</b>	Angle $BAO = x^\circ$ (base $\angle$ s of isos $\Delta$ ) Angle $DAO = y^\circ$ (base $\angle$ s of isos $\Delta$ ) Angle $BCD = 180^\circ - x^\circ - y^\circ$ ( $\angle$ s in opp. segment)		

Qn	Solution	Content/Success Criteria	
<b>14</b>	<b>(a)</b> $(3x+1)(x-5)$  <b>(b)</b> $3(y+1)^2 - 14y - 19$ $= 3(y+1)^2 - 14y - 14 - 5$ $= 3(y+1)^2 - 14(y+1) - 5$ By observation with algebraic expression in part (a), it is observed that $y+1 = x$ . Using answer from part (a), $[3(y+1)+1][(y+1)-5]$ $= (3y+4)(y-4)$	I can factorise quadratic expression in the form $ax^2 + bx + c$ .	
		Content	L
		Complexity	M
		Context	L
		Response Strategy	M
		<b>Assessment Objective</b>	AO1/ AO2

Qn	Solution	Content/Success Criteria	
15	Substitute $x = 2$ and $y = 3$ into equation of the curve $3 = a(2)^2 + b(2) + 2$ $3 = 4a + 2b + 2$ $1 = 4a + 2b$ -----(1) Substitute $x = -1$ and $y = -3$ into equation of the curve $-3 = a(-1)^2 + b(-1) + 2$ $-3 = a - b + 2$ $-5 = a - b$ -----(2)  From (2), $a = -5 + b$ -----(3)  Substitute (3) into (1) $1 = 4(-5 + b) + 2b$ $1 = -20 + 4b + 2b$ $21 = 6b$ $b = 3.5$  Substitute $b = 3.5$ into (3) $a = -5 + 3.5$ $a = -1.5$ Therefore, $a = -1.5$ and $b = 3.5$	I can apply the concept of substituting coordinates of points to form and solve linear equations in two variables.	
		Content	M
		Complexity	L
		Context	L
		Response Strategy	L
		Assessment Objective	AO2

Qn		Solution	Content/Success Criteria	
16	(a)	$P(\text{choosing a red marble}) = 1 - \frac{1}{6} - \frac{7}{12}$ $= \frac{1}{4}$ <p>Let the total number of marbles in the bag be <math>x</math>.</p> $\frac{1}{4}x = 15$ $x = 15 \times 4$ $= 60$	I can find the probability of single and combined events.	
	(b)	<p>Number of blue marbles = <math>\frac{1}{6} \times 60</math></p> $= 10$ $\frac{10}{60} \times \frac{9}{59}$ $= \frac{3}{118}$	Content	L
			Complexity	L
			Context	L
			Response Strategy	L
			Assessment Objective	AO1

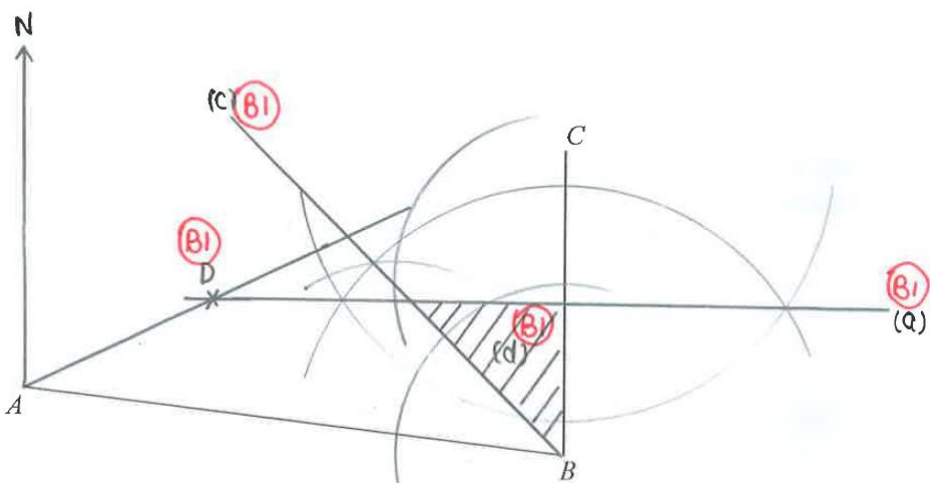
Qn	Solution	Content/Success Criteria		
17	(a)	$\angle BCD = \frac{(5-2) \times 180^\circ}{5}$ $= 108^\circ$ $\angle DCI = \frac{(6-2) \times 180^\circ}{6}$ $= 120^\circ$ $\angle BCI = 360^\circ - 108^\circ - 120^\circ$ ( $\angle$ s at a point) $= 132^\circ$		
		Int. $\angle$ + Ext. $\angle$ = $180^\circ$ Sum of int. $\angle$ s = $(n-2) \times 180^\circ$ Number of sides = $\frac{360^\circ}{\text{Value of 1 ext. } \angle}$		
		Content	L	
		Complexity	L	
		Context	L	
		Response Strategy	L	
		Assessment Objective	AO2	
		(b)	Exterior angle = $180^\circ - 132^\circ$ $= 48^\circ$ Number of sides of polygon = $\frac{360^\circ}{48^\circ}$ $= 7.5$ Since the number of sides is not a positive integer greater than 3, $BC$ and $CI$ cannot be sides of a regular polygon.	
			I can use properties of polygons to determine if a polygon has equal sides.	
			Content	L
			Complexity	L
			Context	L
Response Strategy	M			
Assessment Objective	AO3			
Alternatively, using interior angles of polygon				
$132^\circ = \frac{(n-2) \times 180^\circ}{n}$ $132^\circ n = 180^\circ n - 360^\circ$ $48^\circ n = 360^\circ$ $n = 7.5$ Since the number of sides is not a positive integer greater than 3, $BC$ and $CI$ cannot be sides of a regular polygon.				

Qn		Solution	Content/Success Criteria	
18	(a)	$\frac{1}{2} \times \left( \frac{42}{60} + \frac{63}{60} \right) \times v = 14$ $\frac{7}{8} v = 14$ $v = 16 \text{ (shown)}$ <p>Alternatively,</p> $v = \frac{4}{15} \text{ km/h} \qquad v = \frac{4}{15} \text{ km/min}$ $= \frac{4}{15} \div 60 \text{ km/h} \qquad \text{or} \qquad = \frac{4 \text{ km}}{\frac{15}{60} \text{ h}}$ $= 16 \text{ km/h} \qquad \qquad \qquad = 16 \text{ km/h}$	I can use distance in speed-time graph to find speed.	
			Content	L
			Complexity	M
			Context	L
			Response Strategy	H
			Assessment Objective	AO2
	(b)	$\frac{5}{11} \times 16$ $= 7 \frac{3}{11} \text{ km/h or } 7.27 \text{ km/h (3 sf)}$	I can find unknown speed and acceleration in speed-time graph.	
			Content	L
	(c)	Acceleration = Gradient in last 10 mins $= \frac{0-16}{\frac{10}{60}}$ $= -96 \text{ km/h}^2$	Complexity	L
			Context	L
			Response Strategy	L
			Assessment Objective	AO1

Qn		Solution	Content/Success Criteria	
19	(a)	23	I can apply concepts of mode and range to stem-and-leaf diagram.	
	(b)	$\frac{4}{14} \times 100\% = 28 \frac{4}{7} \%$	Content	L
			Complexity	M
			Context	L
			Response Strategy	M
	(c)	$11 + 31 = 42$ $40 - 31 = 9$	Assessment Objective	AO1/ AO2

Qn		Solution	Content/Success Criteria	
20	(a)	$T_5 = \frac{6}{5^4} - \frac{7}{5^5} = \frac{23}{3125}$	I can recognize and represent pattern by finding an algebraic expression for the $n$ th term. Skills: $5^{n-1} = 5^n \times 5^{-1}$ $5^{-1} = \frac{1}{5}$	
	(b)	$T_n = \frac{n+1}{5^{n-1}} - \frac{n+2}{5^n}$ $= \frac{5(n+1)}{5^n} - \frac{n+2}{5^n}$ $= \frac{5(n+1) - (n+2)}{5^n}$ $= \frac{5n+5-n-2}{5^n}$ $= \frac{4n+3}{5^n} \text{ (shown)}$		
	(c)	$T_1 + T_2 + T_3 + \dots + T_{10}$ $= \left(\frac{2}{1} - \frac{3}{5}\right) + \left(\frac{3}{5} - \frac{4}{5^2}\right) + \left(\frac{4}{5^2} - \frac{5}{5^3}\right) + \dots + \left(\frac{11}{5^9} - \frac{12}{5^{10}}\right)$ $= \frac{2}{1} - \frac{12}{5^{10}}$ $= 2.00 \text{ (to 3 s.f.)}$	Content	L
			Complexity	M
			Context	L
			Response Strategy	M
			Assessment Objective	AO1/ AO2
			Content	L
			Complexity	M
			Context	L
			Response Strategy	H
			Assessment Objective	AO3

Qn		Solution	Content/Success Criteria	
21	(a)	$t + 12 + t - 2$ $= (2t + 10) \text{ minutes}$	I can apply concepts of average speed to find time.	
	(b)	<p>Average speed = <math>\frac{\text{Total distance}}{\text{Total time}}</math></p> $10.5 = \frac{8.75}{\left(\frac{2t + 10}{60}\right)}$ $\frac{2t + 10}{60} = \frac{8.75}{10.5}$ $2t + 10 = 60 \left(\frac{8.75}{10.5}\right)$ $2t + 10 = 50$ <p>Total time = 50 minutes</p> <p><i>Alternatively:</i></p> $10.5 \text{ km/h} = \frac{10.5 \text{ km}}{1 \text{ h}}$ $= \frac{10.5 \text{ km}}{60 \text{ min}}$ $= \frac{7}{40} \text{ km/min}$ <p>Average speed = <math>\frac{\text{Total distance}}{\text{Total time}}</math></p> $\frac{7}{40} = \frac{8.75}{2t + 10}$ $7(2t + 10) = 350$ $14t + 70 = 350$ $14t = 280$ $t = 20$ <p>Time taken for the whole journey = <math>2(20) + 10</math>  <math>= 50 \text{ minutes}</math></p>		
			Content	L
			Complexity	L
			Context	L
			Response Strategy	L
			Assessment Objective	AO1/ AO2

Qn		Solution	
22		 <p><i>*Note: North line at A must be parallel to BC because C is due north of B.</i></p>	
		<b>Content/Success Criteria</b>	
		I can construct perpendicular bisectors, angle bisectors and points with given bearing. I can draw conclusions of the required area after constructing the bisectors.	
	Content	L	
	Complexity	L	
	Context	L	
	Response Strategy	M	
	<b>Assessment Objective</b>	AO1/AO2	

Qn		Solution	Content/Success Criteria	
23	(a)(i)	4.2	I can calculate mean and standard deviation from grouped data using a calculator.	
	(a)(ii)	2.47	Content	L
			Complexity	L
			Context	L
			Response Strategy	L
			<b>Assessment Objective</b>	AO1



	(b)	The mean number of books will increase by 2 while the standard deviation value will remain the same.	I can draw simple inferences from mean and standard deviation values.	
	(c)	SD of class A = 2.47 SD of class B = 3.15 Since the standard deviation value of class A is lesser than class B by 0.68, class A has a smaller spread about the mean and the number of books read by class A is more homogeneous in general.	Content	M
			Complexity	L
			Context	L
			Response Strategy	H
			Assessment Objective	AO3

Qn	Solution		Content/Success Criteria	
24	(a)	$W = 5 \begin{pmatrix} 25 & 32 & 40 \\ 21 & 19 & 32 \end{pmatrix}$ $= \begin{pmatrix} 125 & 160 & 200 \\ 105 & 95 & 160 \end{pmatrix}$	I can solve problems involving sum and product of matrices.	
	(b)			
	(c)			
	(d)			
		$C = \begin{pmatrix} 40 \\ 25 \\ 30 \end{pmatrix}$ $T = \begin{pmatrix} 125 & 160 & 200 \\ 105 & 95 & 160 \end{pmatrix} \begin{pmatrix} 40 \\ 25 \\ 30 \end{pmatrix}$ $= \begin{pmatrix} 15000 \\ 11375 \end{pmatrix}$	Content	L
			Complexity	L
			Context	L
			Response Strategy	M
		The elements in T represent the total amount of school fees paid for weekdays in a week for beginner and advanced students respectively.	Assessment Objective	AO1/ AO2

Qn		Solution	Content/Success Criteria	
25	(a)	$OQ = OS$ (radii of small circle) $\angle OQP = \angle OSR$ (radius $\perp$ tangent) $\angle POQ = \angle ROS$ (vert. opp. $\angle$ s) $\triangle OPQ$ is congruent to $\triangle ORS$ (ASA congruency test)  Alternatively, $OQ = OS$ (radii of small circle) $OP = OR$ (radii of big circle) $\angle PQO = \angle RSO$ (radius $\perp$ tangent) $\triangle OPQ$ is congruent to $\triangle ORS$ (RHS congruency test)	I can determine if two triangles are congruent.	
			Content	L
			Complexity	M
			Context	H
			Response Strategy	H
			Assessment Objective	AO3
	(b)	Let the radius of the small circle be $5r$ . $\angle PQO = \angle RSO$ (radius $\perp$ tangent) Area of one shaded triangle $= \frac{1}{2} \times 5r \times 12r$ $= 30r^2$ Area of big circle $= \pi (13r)^2$ $= 169\pi r^2$ Percentage of shaded region $= \frac{2 \times 30r^2}{169\pi r^2} \times 100\%$ $= \frac{60}{169\pi} \times 100\%$ $= 11.3\%$ (3 s.f.)	I can use ratio and percentage to find area of circles and triangles.	
			Content	L
			Complexity	L
			Context	H
			Response Strategy	M
			Assessment Objective	AO2
OR		Let the radius of the small circle be $r$ . $PQ = \frac{12}{5}r$ $OP = \frac{13}{5}r$ Area of one shaded triangle $= \frac{1}{2} \times r \times \left(\frac{12}{5}r\right)$ $= 1.2r^2$ Area of big circle $= \pi \left(\frac{13}{5}r\right)^2$ $= 6.76\pi r^2$ Percentage of shaded region $= \frac{2 \times 1.2r^2}{6.76\pi r^2} \times 100\%$ $= \frac{2.4}{6.76\pi} \times 100\%$ $= 11.3\%$ (3 s.f.)		