



**HUA YI SECONDARY SCHOOL**  
**PRELIMINARY EXAMINATION 2024**

**4-G3 /**  
**5-G2**

NAME

CLASS

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INDEX  
NUMBER

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**MATHEMATICS**  
**PAPER 1**

**4052/01**

**13 August 2024**  
**2 hour 15 minutes**

Candidates answer on the Question Paper.

# MARKING SCHEME

**For Examiner's  
Use**

**90**

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Setter: Ms Jasmine Tan

[Turn Over

***Mathematical Formulae****Compound interest*

$$\text{Total Amount} = P \left( 1 + \frac{r}{100} \right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$$

1 (a) Calculate  $\frac{-(-11.8) - \sqrt{(-11)^2 - 7 \times 16 \times (-40)}}{2 \times 16}$ .

$$-1.750958 = -1.75 \quad (3 \text{ s.f.}) \text{ ---- B1}$$

Answer ..... [1]

- (b) There are 800 people in an auditorium, correct to the nearest hundred.  
State the minimum number of people that could be in the auditorium at this time.

$$750 \text{ people ---- B1}$$

Answer ..... people [1]

- 2 (a) Express 1400 as the product of its prime factors.

$$1400 = 2^3 \times 5^2 \times 7 \text{ ---- B1}$$

Answer ..... [1]

- (b) Write down the smallest positive integer  $k$  such that  $1400k$  is a perfect cube.

$$1400k = 2^3 \times 5^2 \times 7 \times k = 2^3 \times 5^2 \times 7 \times (5 \times 7^2)$$

$$k = 245 \text{ ---- B1}$$

Answer  $k =$  ..... [1]

- (c)  $n$  is a number between 300 and 400.

The highest common factor of  $n$  and 1400 is 35.

Find the largest possible value for  $n$ .

$$n = 2^? \times 5 \times 7 \times ?$$

$$1400 = 2^3 \times 5^2 \times 7$$

---


$$\text{HCF} = 5 \times 7 = 35$$

By guess-and-check,  $n = 5 \times 7 \times 11 = 385$  --- B2

**[Condition: largest possible value between 300 – 400]**

Answer  $n =$  ..... [2]

- 3 (a) Simplify  $y^0 \div 9x^{-2} \times x^7$ .

$$y^0 \div 9x^{-2} \times x^7 = 1 \times \frac{x^2}{9} \times x^7 \quad \text{--- M1}$$

$$= \frac{x^9}{9} \quad \text{--- A1}$$

Answer ..... [2]

- (b) Simplify  $(81a^{12})^{\frac{5}{4}}$ .

$$(81a^{12})^{\frac{5}{4}} = (\sqrt[4]{81})^5 \times a^{12 \times \frac{5}{4}}$$

$$= 243a^{15} \quad \text{--- B1}$$

Answer ..... [1]

- 4 (a) Express as a single fraction in its simplest form  $\frac{18b^7}{5c^2} \div \frac{3b^4}{81}$ .

$$\frac{18b^7}{5c^2} \div \frac{3b^4}{81} = \frac{18b^7}{5c^2} \times \frac{81}{3b^4}$$

$$= \frac{486b^3}{5c^2} \quad \text{---} \left[ \text{M1 for } \frac{486}{5}, \text{ A1 for full answer} \right]$$

Answer ..... [2]

- (b) Use the laws of indices to show that  $6^4 \times 100 + 116 \times 36^2$  can be expressed as a single power of six.

Answer ..... [2]

$$6^4 \times 100 + 116 \times 36^2 = 6^4 \times 100 + 116 \times 6^4$$

$$= 6^4 [100 + 116] \quad \text{--- M1}$$

$$= 6^4 \times 216$$

$$= 6^7 \quad \text{--- A1 (shown)}$$

- 5 In a greenhouse, the estimated number of flowering plants increased from 4100 in January 2024 to 4980 in June 2024. The number increased by  $c\%$  every month.  
Find the value of  $c$ .

$$\begin{aligned}
 4980 &= P \left( 1 + \frac{r}{100} \right)^n \\
 4980 &= 4100(1 + 0.01c)^5 \quad \text{--- M1} \\
 \sqrt[5]{\frac{249}{205}} &= 1 + 0.01c \\
 c &= \frac{\sqrt[5]{\frac{249}{205}} - 1}{0.01} \quad \text{--- M1} \\
 &= 3.965 = 3.97 \quad (3\text{s.f.}) \quad \text{--- A1}
 \end{aligned}$$

Answer  $c = \dots\dots\dots$  [3]

- 6 Kyle runs a tennis club. 54 of the members are adults and 31 are children.  
His aim is that **at least** 60% the members should be children.  
Form an inequality to find the smallest number of children that Kyle would still need to recruit achieve his aim.

Let  $c$  be the no. of children needed to join the club.

$$\begin{aligned}
 \frac{31+c}{54+31+c} &\geq 60\% \quad \text{--- M1 (forms inequality)} \\
 31+c &\geq 0.6(85+c) \\
 31+c &\geq 51+0.6c \\
 c-0.6c &\geq 20 \quad \text{--- M1 (isolate unknown)} \\
 0.4c &\geq 20 \\
 c &\geq 50 \\
 \therefore \text{Smallest number} &= 50 \quad \text{--- A1}
 \end{aligned}$$

Answer  $\dots\dots\dots$  children [3]

- 7 A car travels at an average speed of 74.5 km/h for 2.25 hours.

(a) Convert 75 km/h to m/s.

$$\frac{75\,000}{3600} = 20\frac{5}{6} = 20.8 \text{ m/s} \quad \text{--- B1 (Accept either 1 form)}$$

Answer ..... m/s [1]

(b) By rounding the numbers correct to 1 significant figure, find an estimate of the distance travelled by the car. Show your working clearly.

$$\begin{aligned} \text{Distance} &= 74.5 \times 2.25 = 70 \times 2 \quad \text{--- M1 (must show rounding to 1 s.f. each)} \\ &= 140 \text{ km} \quad \text{--- A1} \end{aligned}$$

Answer ..... km [2]

(c) Without doing any calculation, explain why the actual distance travelled by the car is greater than the answer to (b).

Answer [1]

Both the average speed and time is greater than the 1 significant figure rounded values of speed and time respectively. Thus, the actual distance travelled is greater than the answer in **part (b)**.

----- B1

- 8 Isha has written down five numbers.

The mean of these numbers is 13.2, the median is 12 and the mode is 7.

The largest number is three times the smallest number.

Find the five numbers in **ascending** order.

7	7	12	y	21
<i>Smallest</i>	<i>Median</i>			<i>largest</i>
Mean = 13.2	<div style="border: 1px dashed black; padding: 5px; color: magenta;">           Any 3 consecutive values correct → B1            ALL 5 values correct → Full mark, B2         </div>			
$\frac{\text{Sum}}{5} = 13.2$				
Sum = 66 , so y = 19.				

Answer ..... [2]

**9** Factorise completely

(a)  $2p^4 - 32s^4$ ,

$$\begin{aligned}
 2p^4 - 32s^4 &= 2 \left[ (p^2)^2 - (4s^2)^2 \right] \quad \text{--- M1 [factorise by HCF]} \\
 &= 2 \left[ (p^2 + 4s^2)(p^2 - 4s^2) \right] \quad \text{--- M1 [factorise by } a^2 - b^2 \text{]} \\
 &= 2 \left[ (p^2 + 4s^2)(p + 2s)(p - 2s) \right] \quad \text{--- A1 [factorise by } a^2 - b^2 \text{ again]}
 \end{aligned}$$

Answer ..... [3]

(b)  $12cd - 9cx + 6xy - 8dy$ .

$$\begin{aligned}
 &3c(4d - 3x) + 2y(3x - 4d) \\
 &= 3c(4d - 3x) - 2y(4d - 3x) \quad \text{--- M1 [Factorise out -ve sign]} \\
 &= (3c - 2y)(4d - 3x) \quad \text{--- A1}
 \end{aligned}$$

Answer ..... [2]

**10** (a) Express  $9 - 8x + x^2$  in the form  $a + (x + b)^2$ . Find the value of  $a$  and of  $b$ .

$$\begin{aligned}
 9 - 8x + x^2 &= (x - 4)^2 + 9 - 16 = -7 + (x - 4)^2 \\
 a &= -7 \quad \text{--- B1}, \quad b = -4 \quad \text{--- B1}
 \end{aligned}$$

Answer  $a =$  ..... [1] $b =$  ..... [1](b) Explain why when  $x = 4$ , the expression  $9 - 8x + x^2$  has its minimum value.

Answer ..... [1]

For any perfect square, the smallest value is always equal to zero or greater than zero.

When  $x = 4$ ,  $(x - 4)^2 = 0$  thus,  $(x - 4)^2 - 7 = -7 < 0$  and the coefficient of  $x^2$  is positive, implying that the quadratic expression will have a minimum value at  $x = 4$ . ----- **B1**

- 11 Solve the equation  $5 + 2x = \frac{20}{1+x}$ .

$$5 + 2x = \frac{20}{1+x}$$

$$5 + 5x + 2x + 2x^2 = 20$$

$$2x^2 + 7x - 15 = 0 \quad \text{--- M1}$$

$$(2x-3)(x+5) = 0 \quad \text{--- M1}$$

$$x = 1.5 \quad \text{or} \quad x = -5 \quad \text{--- A1} \quad [\text{BOTH ans correct}]$$

*Answer*  $x = \dots\dots\dots \text{ or } \dots\dots\dots$  [3]

- 12 The points  $(4, 20)$  and  $(10, -4)$  satisfy the curve given by the equation  $y = ax^2 + bx - 4$ .

Use an algebraic method to determine the values of  $a$  and  $b$ .

$$20 = a(4)^2 + b(4) - 4$$

$$24 = 16a + 4b$$

$$6 = 4a + b \quad \text{--- Eqn(1)}$$

$$-4 = a(10)^2 + b(10) - 4$$

$$-4 = 100a + 10b - 4$$

$$100a = -10b$$

$$10a + b = 0 \quad \text{--- Eqn(2)} \quad \text{--- M1} \quad [\text{any 1 eqn correct}]$$

**\*Solve by either elimination or substitution ----- Award M1**

$$a = -1, \quad b = 10 \quad \text{--- A2}$$

*Answer*  $a = \dots\dots\dots, b = \dots\dots\dots$  [4]

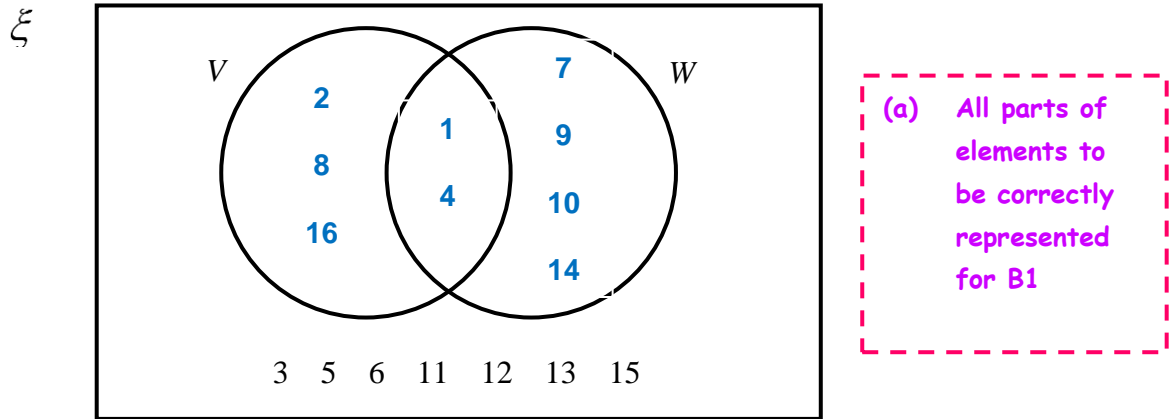


13  $\xi = \{\text{integers } x : 1 \leq x \leq 16\}$

$V = \{1, 2, 4, 8, 16\}$

$W = \{1, 4, 7, 9, 10, 14\}$

Some of the information is shown on the Venn diagram.



- (a) Complete the Venn diagram by representing all the elements in the given sets.

Answer in above Venn diagram

[1]

- (b) Describe the elements of Set  $V$ .

Answer

[1]

Elements of Set  $V$  are the factors of 16. ----- B1

.....

- (c) List the elements contained in the set  $V' \cap W$ .

Answer  $\{7, 9, 10, 14\}$  ----- B1 [1]

- (d) Find the number of elements in  $(V \cap W') \cup (V \cup W)'$ .

Answer 10 ----- B1 [1]

- (e) Use one of the symbols below to complete each statement.

$= \quad \emptyset \quad \subset \quad \not\subset \quad \notin \quad \in \quad \xi$

(i)  $\{2, 16\}$  .....  $V$

(ii) 13 .....  $W$

Answer (e) (i) ...  $\subset$  ... [1]

(ii) ....  $\notin$  ... [1]

- 14 In grocery mart  $G$ , water costs \$1.80 per litre, fresh milk costs \$2.90 per litre and juice costs \$2.30 per litre.

In grocery mart  $H$ , water costs \$0.20 more per litre, fresh milk costs \$0.40 less per litre and juice costs \$0.10 less per litre.

This information can be represented by the matrix  $\mathbf{Q} = \begin{matrix} & \begin{matrix} G & H \end{matrix} \\ \begin{matrix} W \\ M \\ J \end{matrix} & \begin{pmatrix} 1.8 & 0.2 \\ 2.9 & -0.4 \\ 2.3 & -0.1 \end{pmatrix} \end{matrix}$ .

- (a) Rayyen and Zinnie went shopping together.

Rayyen bought 4 litres of water, 2 litres of milk and 3 litres of juice.

Zinnie bought 3 litres of water and 4 litres of juice.

Represent their purchases in a  $2 \times 3$  matrix  $\mathbf{P}$ .

$$\mathbf{P} = \begin{matrix} & \begin{matrix} W & M & J \end{matrix} \\ \begin{matrix} Rayyen \\ Zinnie \end{matrix} & \begin{pmatrix} 4 & 2 & 3 \\ 3 & 0 & 4 \end{pmatrix} \end{matrix} \quad \text{--- B1}$$

Answer  $\mathbf{P} = \dots\dots\dots$  [1]

- (b) Evaluate the matrix  $\mathbf{R} = \mathbf{PQ}$ .

$$\mathbf{R} = \begin{matrix} & \begin{matrix} W & M & J \end{matrix} \\ \begin{matrix} Rayyen \\ Zinnie \end{matrix} & \begin{pmatrix} 4 & 2 & 3 \\ 3 & 0 & 4 \end{pmatrix} \end{matrix} \begin{pmatrix} 1.8 & 0.2 \\ 2.9 & -0.4 \\ 2.3 & -0.1 \end{pmatrix}$$

$$= \begin{pmatrix} 19.9 & -0.3 \\ 14.6 & 0.2 \end{pmatrix} \quad \text{--- B1 [Any 2 correct values, B1, all correct B2]}$$

Answer  $\mathbf{R} = \dots\dots\dots$  [2]

- (c) State what the elements in the second column of matrix  $\mathbf{R}$  represent.

Answer [1]

They represent the difference in the total spendings of Rayyen and of Zinnie at grocery mart  $G$  and  $H$  respectively for water, fresh milk and juice. ----- B1

- (d) Rayyen shopped in grocery mart  $H$ . He got a discount coupon that entitled him to a discount of 15%. How much did he pay altogether for his items?

$$\text{Amount Rayyen spent} = 19.90 - 0.30 = \$19.60 \quad \text{--- M1}$$

$$\text{Amount to pay after 15\% discount} = 19.60 \times 85\% = \$16.66 \quad \text{--- A1}$$

Answer \$ \dots\dots\dots [2]

**15** In a regular polygon, the ratio of an interior angle : exterior angle = 14 : 1.

(a) Find the number of sides of the polygon.

$$\begin{array}{l} 15 \text{ units} \text{ ----- } 180^\circ \\ 1 \text{ unit} \text{ ----- } 12^\circ \text{ --- } M1 \end{array}$$

$$\text{Sum of exterior angles} = 360^\circ$$

$$\begin{array}{l} \text{No. of sides} = \frac{360^\circ}{12} \text{ --- } M1 \\ = 30 \text{ sides} \text{ --- } A1 \end{array}$$

*Answer* ..... sides [3]

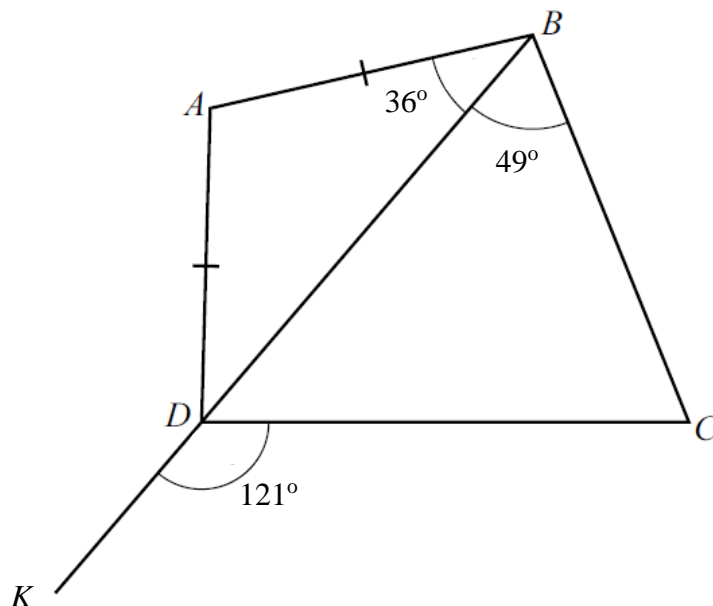
(b) Find the sum of the interior angles of the polygon.

$$\text{Sum of interior angles} = (n - 2)180^\circ$$

$$\begin{array}{l} \text{No. of sides} = (30 - 2)180^\circ \\ = 5040^\circ \text{ --- } A1 \text{ (ECF)} \end{array}$$

*Answer* .....° [1]

- 16 In the diagram,  $BDK$  is a straight line and  $AB = AD$ .  
 Angle  $ABD = 36^\circ$ , angle  $CBD = 49^\circ$  and angle  $CDK = 121^\circ$ .



Explain why it is possible to draw a circle that passes through the points  $A$ ,  $B$ ,  $C$  and  $D$ .  
 Give reasons for each step of your working.

Answer

[3]

If a circle can pass through all points  $A$ ,  $B$ ,  $C$  and  $D$ , then

**Angle  $DAB$  + Angle  $DCB$  =  $180^\circ$  (angles in opp. segment)**

**CHECK:**

$$\angle ADB = \angle ABD = 36^\circ \quad (\text{isos. } \Delta)$$

$$\angle DAB = 180^\circ - 36^\circ - 36^\circ = 108^\circ \quad (\text{sum of angles in } \Delta) \quad \text{--- M1}$$

$$\angle BDC = 180^\circ - 121^\circ = 59^\circ \quad (\text{adj. angles on str. line})$$

$$\angle DCB = 180^\circ - 59^\circ - 49^\circ = 72^\circ \quad (\text{sum of angles in } \Delta) \quad \text{--- M1}$$

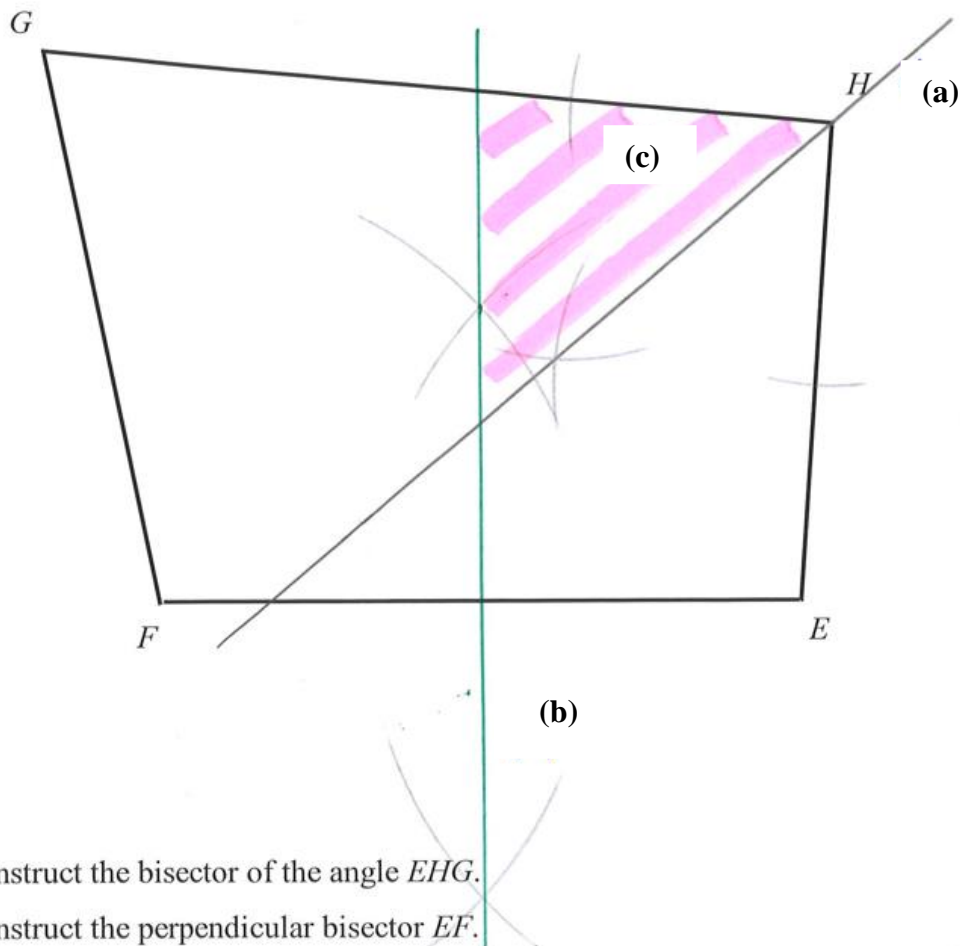
$$\therefore \angle DAB + \angle DCB = 108^\circ + 72^\circ = 180^\circ$$

**(angles in opp. segment) ----- A1**

Since the above two angles add up to  $180^\circ$ , it means that it will be possible to draw a circle that passes through all points  $A$ ,  $B$ ,  $C$  and  $D$ .

**\*\*Any step without/ supported with incorrect reason more than once, DEDUCT 1 mark from overall point score for Q16.**

- 17 The diagram represents a plot of land,  $EFGH$ , which is to be used for an observatory.



- (a) Construct the bisector of the angle  $EHG$ . [1]
- (b) Construct the perpendicular bisector  $EF$ . [1]
- (c) A café is to be built in the observatory, nearer to  $E$  than to  $F$  and nearer to  $GH$  than to  $EH$ .  
Shade the region where the café is to be built. [1]

**GUIDANCE:**

**\*\* Must show construction arcs clearly for both parts (i) and (ii) to award B1 mark EACH.**

**\*\* If parts (i) and/ or (ii) wrong, but able to shade based on descriptions, award A1 (ECF).**

- 18 (a) A cargo ship has an average fuel consumption of 0.000 892 kilometres per litre.  
Write this consumption in litres per kilometre.

$$0.000\,892\text{ km} \rightarrow 1l$$

$$1\text{ km} \rightarrow \frac{1}{0.000\,892}$$

$$= 1121.076$$

$$= 1120l \quad (3 \text{ s.f.}) \quad \text{--- } B1$$

Answer ..... l / km [1]

- (b) A model of another cargo ship is made to a scale of 1 : 60.  
The length of this model cargo ship is 550 cm.

- (i) Find the actual length of this cargo ship in metres.

$$1\text{ cm} : 0.6\text{ m}$$

$$550\text{ cm} : 330\text{ m} \quad \text{--- } B1$$

Answer ..... m [1]

The capacity of the fuel tanks in the model cargo ship is 7.75 litres.

- (ii) Find the actual capacity of the fuel tanks of the cargo ship.  
Express your answer in standard form.

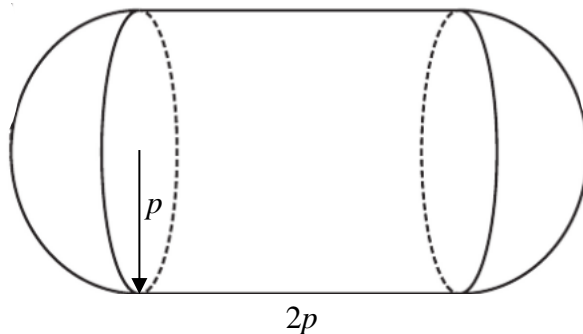
$$\frac{V_1}{V_2} = \left( \frac{l_1}{l_2} \right)^3$$

$$\frac{7.75}{V_{ship}} = \left( \frac{1}{60} \right)^3 \quad \text{--- } M1$$

$$V_{ship} = 1.674 \times 10^6 l \quad \text{--- } A1$$

Answer ..... l [2]

- 19 A composite solid is made from a cylinder and two hemispheres.  
The cylinder has radius  $p$  and length  $2p$ , while the hemispheres have radius  $p$ .



The total surface area of the solid is six times the total surface area of a cone with radius  $p$  and slant height  $l$ .

Find  $l$  in terms of  $p$ .

Since cone is a **SOLID (i.e. closed) cone**,

$$\text{Total S.A. of solid} = 6 \times \text{Total S.A. of cone}$$

$$4\pi r^2 + 2\pi rh = 6(\pi rl + \pi r^2)$$

$$4\pi p^2 + 2\pi p(2p) = 6(\pi pl + \pi p^2) \quad \text{--- M1 [forms equation to connect S.A.]}$$

$$8\pi p^2 - 6\pi p^2 = 6\pi pl \quad \text{--- M1 [isolate } l \text{ term]}$$

$$l = \frac{2\pi p^2}{6\pi p}$$

$$= \frac{p}{3} \quad \text{--- A1}$$

Answer  $l = \dots\dots\dots$  [3]

- 20** Gino can paint 6 fence panels in 4 hours, while Danish can paint 7 fence panels in 5 hours. Gino and Danish work together to paint a total of 21 panels.

If they continue to paint at the same rate, how long will it take them to paint 21 panels?  
Give your answer in hours and minutes, correct to the nearest minute.

For Gino,  $4 \text{ h} \rightarrow 6 \text{ panels}$   
 $1 \text{ h} \rightarrow 1.5 \text{ panels}$

For Danish,  $5 \text{ h} \rightarrow 7 \text{ panels}$   
 $1 \text{ h} \rightarrow 1.4 \text{ panels}$

So in 1 hour, both will finish 2.9 panels ----- M1

With 2 people,  $2.9 \text{ panels} \rightarrow 1 \text{ h}$   
 $21 \text{ panels} \rightarrow \frac{21}{2.9}$  --- M1 [ECF]  
 $= 7.241379 \text{ h} = 7 \text{ h } 14 \text{ min}$  --- A1

**\*ALTERNATIVE acceptable answer**

- Accept also rounded value of 7h 15 min, since some students considered that the work can only be completed after 7h 14 min 29 sec.

*Answer* ..... h ..... min [3]



**21** A fitness centre has 16 employees.

One of the 16 employees is selected at random.

The probability that it is a woman working part time is  $\frac{1}{8}$ .

Two of the 16 employees are selected at random.

The probability that they are both men working full time is  $\frac{1}{8}$ .

Complete the table of information below about the 16 employees of the fitness centre.

Show all supporting calculations clearly.

*Answer*

[4]

	<i>Part-time employees</i>	<i>Full-time employees</i>
<i>Women</i>	$a = 2$ ---- B1	5
<i>Men</i>	3 ---- B1	$b = 6$ ---- A1

$$\frac{a}{16} = \frac{1}{8} \Rightarrow a = 2$$

$$P(\text{both men full time}) = \frac{1}{8}$$

$$\frac{b}{16} \times \frac{b-1}{15} = \frac{1}{8}$$

$$b^2 - b = 30$$

$$b^2 - b - 30 = 0$$

$$(b-6)(b+5) = 0 \quad \text{--- M1}$$

$$b = 6 \quad \text{or} \quad b = -5 \text{ (N.A.)}$$

- 22 A librarian wants to find out how much time patrons spend at the library in a week. He uses the questionnaire.

*How many hours do you spend at our library in a week (including weekends)?*

*Please tick one box.*

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*1 – 2*

*3 – 4*

*5 – 7*

*over 7*

List **two** ways to improve the questionnaire.

*Answer*

[2]

**\*Award full marks for any 2 logical improvements for questionnaire.**

### **List of possible suggestions**

- The 3<sup>rd</sup> option has a duration of 2 hours resulting in the options, which is different from the other options. Each option to contain a duration that is consistent.
- The options are not continuous, as there are no options for patrons who may spend 2-3 hours or 4-5 hours. Options can be rephrased in the idea of inequality. E.g. Less than 2 hours, 2 or more hours but less than 3 hours.
- Add in the option of 0 – 1 hour as there may be patrons who spend lesser than 1 hour.
- Rephrase the question to:

***On average***, how many hours do you spend at our library in a week (including weekends)?

This would make the question more accurately phrased as patrons do not always spend the same amount of time in the library during every visit.

23  $K$  is the point  $(7, -8)$  and  $L$  is the point  $(x, y)$ .

The gradient of the line  $KL$  is  $\frac{2}{3}$ .

Maverick claims that it is possible to express  $x$  in terms of  $y$ , such that  $x = a + by$ , where  $a$  and  $b$  are constants.

Explain why Maverick is correct.

*Answer*

[2]

$$\text{Gradient } KL = \frac{y - (-8)}{x - 7}$$

$$\frac{2}{3} = \frac{y + 8}{x - 7}$$

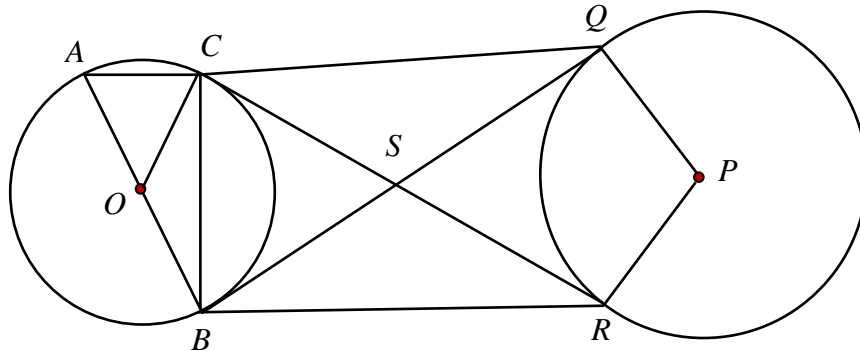
$$2x - 14 = 3y + 24 \quad \text{--- } M1$$

$$x = \frac{3y + 38}{2}$$

$$x = 19 + \frac{3}{2}y \quad \text{--- } A1$$

We see that it is possible to express the equation in the form  $x = a + by$ , where  $a$  and  $b$  are constants. Thus, he is correct.

- 24  $A, B$  and  $C$  are points on the circle with centre  $O$ .  
 $Q$  and  $R$  are points on the circle with centre  $P$ .  
 $CSR$  and  $BSQ$  are tangents on to both circles.  
 $AOB$  is a straight line.



- (a) Show that triangle  $CQS$  is congruent to triangle  $BRS$ .  
 Give a reason for each statement you make.

Answer

[3]

Since  $CSR$  and  $BSQ$  are tangents on both circles, applying the property of tangents from external points,  $CS = BS$  and  $QS = RS$ . ----- M1

$$\angle CSQ = \angle BSR \quad (\text{vert. opp. } \angle\text{s}) \quad \text{----- M1}$$

By SAS property,  $\triangle CQS \equiv \triangle BRS$  ----- A1

**\*\*Any missing reason, deduct 1 mark from overall mark scored for Q19.**

24 (b) Angle  $ABC = y^\circ$

Find, in terms of  $y$ ,

(i) angle  $BAC$ ,

$$\angle ACB = 90^\circ \quad (\text{angle in semicircle})$$

$$\angle BAC = 180^\circ - 90^\circ - y^\circ \quad (\text{angle sum of triangle})$$

$$= (90 - y)^\circ \quad \text{--- B1}$$

Answer .....<sup>o</sup> [1]

(ii) angle  $QPR$ .

$$\angle OCS = \angle OBS = 90^\circ \quad (\text{tan} \perp \text{rad.})$$

$$\angle PSR = \angle PQS = 90^\circ \quad (\text{tan} \perp \text{rad.})$$

$$\angle CSB = \angle RSQ \quad (\text{vert. opp. } \angle\text{s})$$

Quad.  $PQRS$  similar to Quad.  $OBSC$

Any 2 correct statements to  
conclude 2 quads are similar  
→ Award M1

\*Reasons optional

$$\angle OCB = \angle OBC = y^\circ \quad (\text{isos. } \Delta)$$

$$\angle QPR = \angle BOC = (180 - 2y)^\circ \quad \text{--- A1}$$

\*ALTERNATIVE acceptable answer

$$\bullet \angle BOC = (180 - 2y)^\circ \quad \text{--- B1}$$

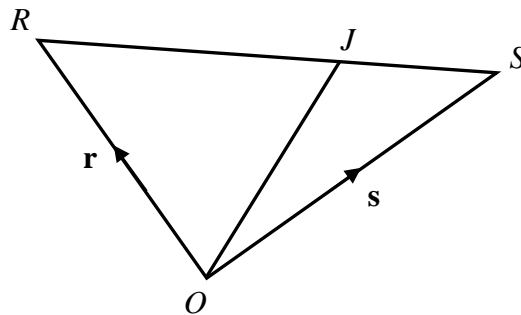
- Award next **B1 mark** for any correct supporting explanation to arrive at above answer.

Answer .....<sup>o</sup> [2]

25  $ORS$  is a triangle.

$J$  is a point on  $RS$  such that  $RJ : JS = 3 : 2$ .

$\overrightarrow{OR} = \mathbf{r}$  and  $\overrightarrow{OS} = \mathbf{s}$ .



(a) Show that  $\overrightarrow{OJ} = \frac{1}{5}(2\mathbf{r} + 3\mathbf{s})$ .

Answer

[2]

$$\overrightarrow{RS} = -\mathbf{r} + \mathbf{s} \quad \text{--- M1}$$

$$\overrightarrow{OJ} = \overrightarrow{OR} + \overrightarrow{RJ}$$

$$= \mathbf{r} + \frac{3}{5}\overrightarrow{RS}$$

$$= \mathbf{r} + \frac{3}{5}(\mathbf{s} - \mathbf{r}) \quad \text{--- M1}$$

$$= \frac{2}{5}\mathbf{r} + \frac{3}{5}\mathbf{s} = \frac{1}{5}(2\mathbf{r} + 3\mathbf{s}) \quad (\text{shown})$$

(b)  $X$  is a point such that  $\overrightarrow{RX} = \frac{1}{5}(\mathbf{r} + 9\mathbf{s})$ .

Explain why  $O$ ,  $J$  and  $X$  lie on a straight line.

Answer

[2]

$$\overrightarrow{RX} = \frac{1}{5}(\mathbf{r} + 9\mathbf{s})$$

$$\overrightarrow{OX} - \overrightarrow{OR} = \frac{1}{5}\mathbf{r} + \frac{9}{5}\mathbf{s}$$

$$\overrightarrow{OX} = \frac{1}{5}\mathbf{r} + \frac{9}{5}\mathbf{s} + \mathbf{r}$$

$$= \frac{6}{5}\mathbf{r} + \frac{9}{5}\mathbf{s}$$

$$\overrightarrow{OX} = 3\left[\frac{2}{5}\mathbf{r} + \frac{3}{5}\mathbf{s}\right] \quad \text{--- M1}$$

$$\overrightarrow{OX} = 3\overrightarrow{OJ}$$

Since the vectors  $\overrightarrow{OX}$  and  $\overrightarrow{OJ}$  **differ by a constant of 3** and have a **common point  $O$** , it means that the **vectors are parallel**.

This further implies that  $OX$  and  $OJ$  are parallel lines. ----- A1

Thus, the points  $O$ ,  $J$  and  $X$  lie on the same straight line.

~ END OF PAPER ~