

ANDERSON SECONDARY SCHOOL
Preliminary Examination 2024
Secondary Four Express and Five Normal



CANDIDATE NAME:

CLASS:

INDEX NUMBER:

MATHEMATICS

4052/01

Paper 1

14 Aug 2024

2 hours 15 min

0800 – 1015 h

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid/tape.

Answer **all** the questions.

If working is needed for any question it must be **neatly and clearly** shown in the space below the question.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 90.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

- 1 Expand and simplify $a(5a - 2b)(5a + 2b)$.

Answer [2]

- 2 Given that $3^x + 3^{x+2} = 90$, find x .

Answer $x =$ [2]

3 Simplify $\frac{2x^2 + 4xy - 3x - 6y}{2x^2 + xy - 6y^2}$.

Answer [3]

- 4 Two integers, A and B , can be written as products of prime factors.

$$A = 3^{m+2} \times 7 \qquad B = q \times 3^m \times 7$$

The lowest common multiple (LCM) of A and B is $3^3 \times 5 \times 7$.

- (a) Write down the value of m and q .

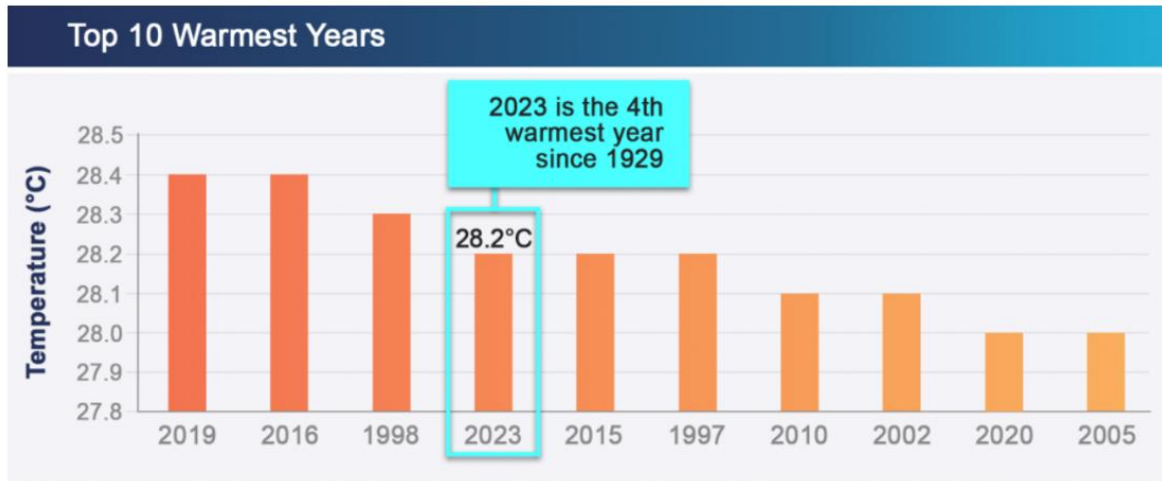
Answer $m = \dots\dots\dots$ [1]

$q = \dots\dots\dots$ [1]

- (b) Find the highest common factor of A and B .

Answer $\dots\dots\dots$ [1]

- 5 Below is a graph from the Annual Climate Singapore 2023, which shows the top 10 warmest years in Singapore since 1929.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

.....

.....

.....

.....[2]

- 6 (a) Simplify $\left(\frac{y^9}{27x^{-6}}\right)^{\frac{2}{3}}$, leaving your answer in positive index.

Answer [2]

- (b) Given $\frac{25}{125^{2-x}} = 5^y$, express x in terms of y .

Answer [2]

- 7 (a) Factorise $9x^2 + 24xy + 16y^2$ completely.

Answer [1]

- (b) Hence factorise $144a^8 - (9a^8 + 24a^4y + 16y^2)$ completely.

Answer [2]

- 8** The intensity of a sound detected by a receiver is inversely proportional to the square of the distance of the receiver from the source of the sound.

When the distance is r metres, the intensity of the sound detected is q decibels.

When the distance is reduced by 40%,

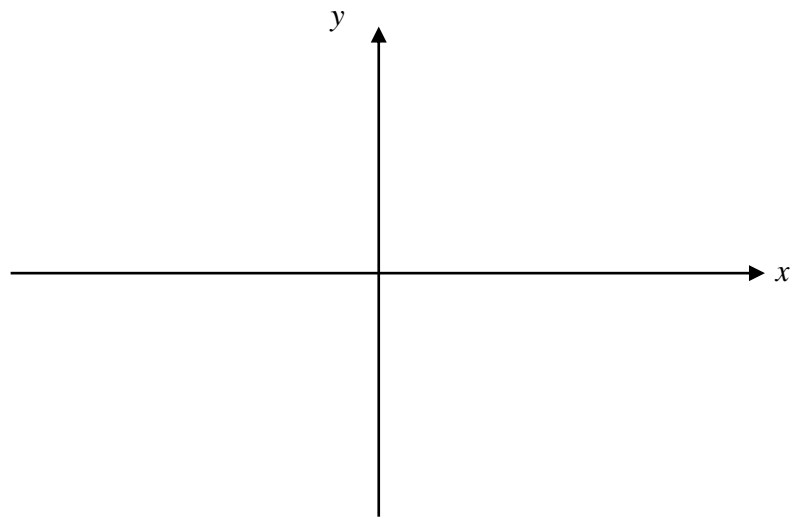
- (a) find the intensity of the sound detected, in terms of q .

Answer decibels [2]

- (b) calculate the percentage difference in the intensity of the sound detected.

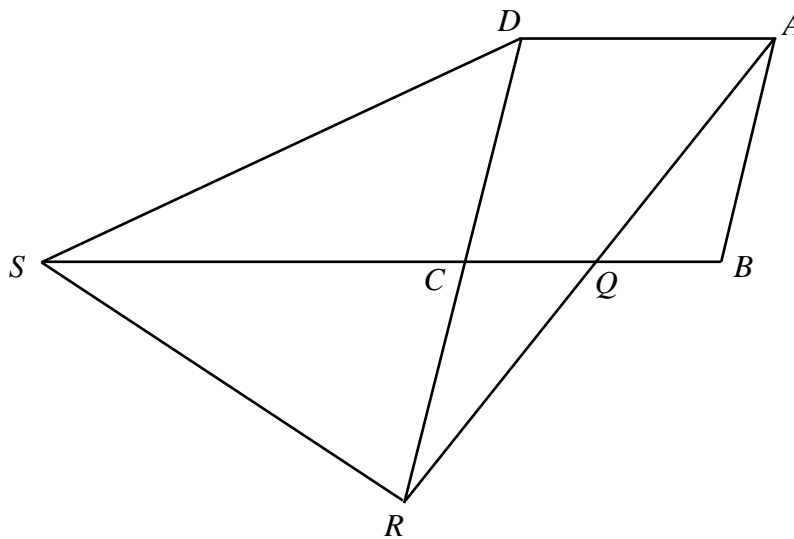
Answer % [1]

- 9 Sketch the graph of $y = 3^{-x}$ on the axes below.
Indicate clearly the coordinates of the points where the graph crosses the axes.



[1]

- 10 $ABCD$ is a rhombus and $RD = CS$.
 SCB , DCR and AQR are straight lines.



Prove that triangles SCD and RDA are congruent.

.....

[3]

- 11** By expressing $x^2 - 10x - 2$ in the form $(x - h)^2 + k$, where h and k are constants, write down the coordinates of the minimum point of the graph of $y = x^2 - 10x - 2$.

Answer (..... ,) [2]

- 12** $\xi = \{(x, y) : x \text{ and } y \text{ are integers, } -2 \leq x < 1 \text{ and } -1 \leq y \leq 1\}$
 $P = \{(x, y) : x^2 + y^2 < 2\}$
 $Q = \{(x, y) : xy < 0\}$

- (a) Find the number of elements in the universal set ξ .

Answer $n(\xi) = \dots\dots\dots$ [1]

- (b) List the elements of the following in set notation.

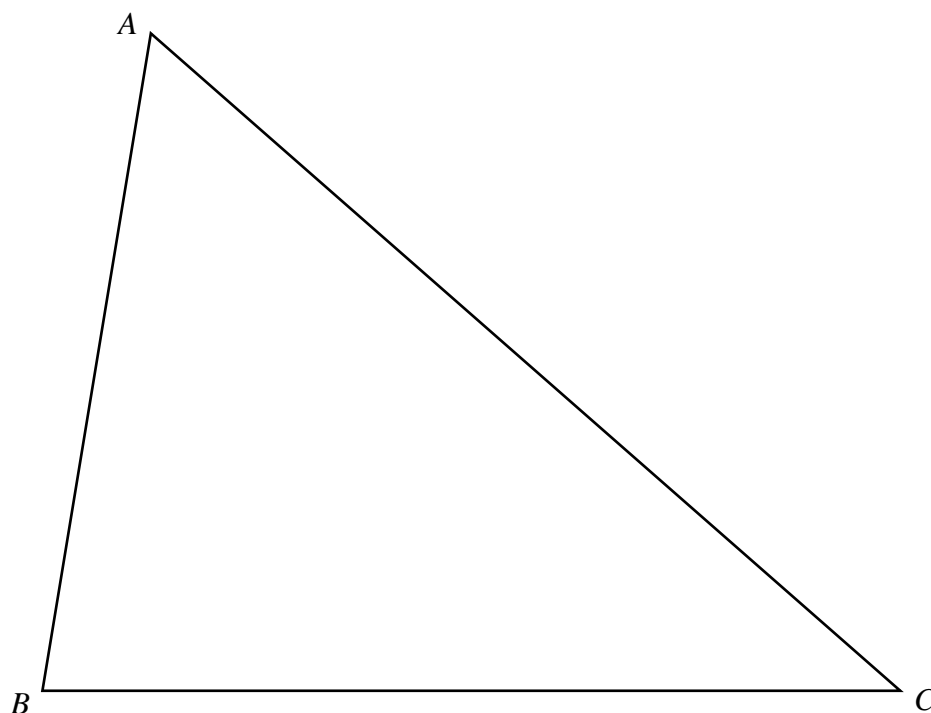
- (i) P

Answer $P = \{ \dots\dots\dots \}$ [1]

- (ii) Q

Answer $Q = \{ \dots\dots\dots \}$ [1]

- 13 The diagram below shows a triangle ABC .



- (a) Construct the perpendicular bisector of AC . [1]
- (b) Construct the bisector of angle BCA . [1]
-

14 $W = \{0, \{0\}\}.$

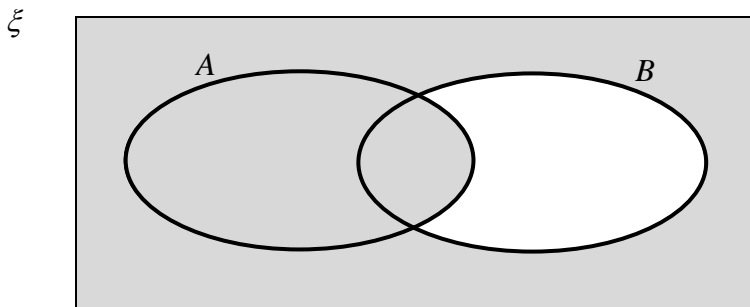
(a) Find $n(W)$.

Answer $n(W) = \dots\dots\dots$ [1]

(b) List all possible subsets of W .

Answer $\dots\dots\dots$ [1]

(c) Write down the set represented by the following shaded region.

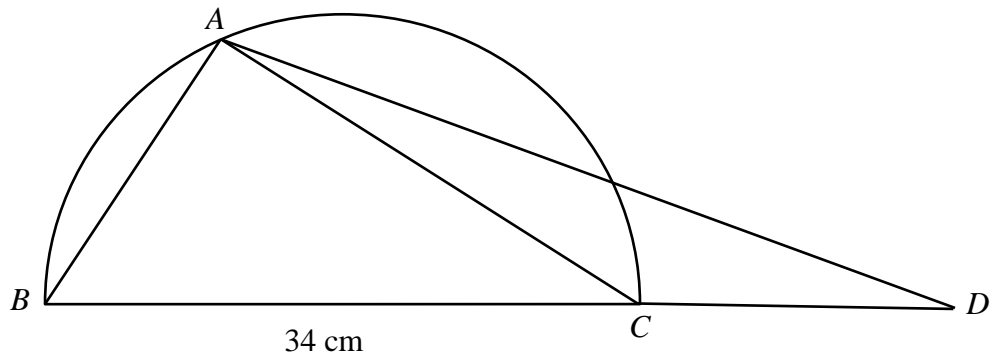


Answer $\dots\dots\dots$ [1]

15 Solve the inequality $x - 3 \leq \frac{5-x}{3} < \frac{x+1}{2}.$

Answer $\dots\dots\dots$ [3]

- 16** The diagram shows a semi-circle BAC , with diameter $BC = 34$ cm.
 AD and BCD are straight lines.



Given that $\cos \angle ABC = \frac{8}{17}$, without solving for angle ABC and angle ACD , find the exact value of $\cos \angle ACD$.

Answer [4]

- 17 The sequence $T_1, T_2, T_3, T_4, \dots$ has the following first four terms.

$$T_1 = 2^2 - 1 = 3$$

$$T_2 = 3^2 - 2 = 7$$

$$T_3 = 4^2 - 3 = 13$$

$$T_4 = 5^2 - 4 = 21$$

- (a) Find the 7th term of the sequence, T_7 .

Answer $T_7 = \dots\dots\dots$ [1]

- (b) Write the expression for the n th term of the sequence.

Answer $T_n = \dots\dots\dots$ [1]

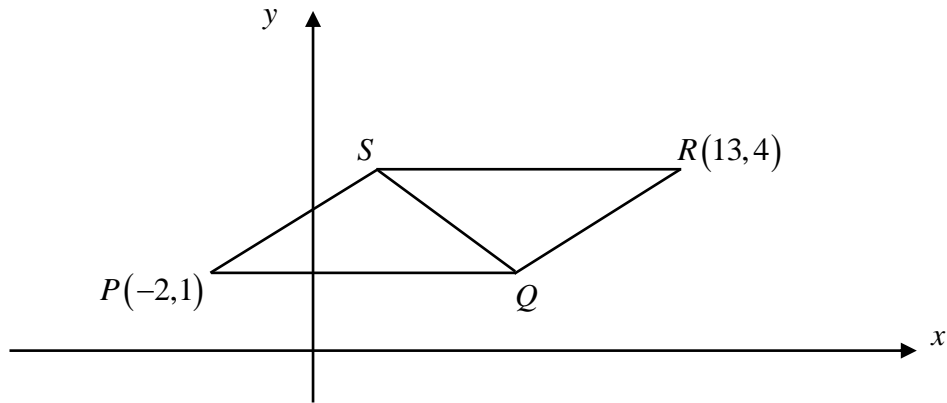
- (c) Kelvin claims that the difference between 2 consecutive terms $T_{n+1} - T_n$, where n is a positive integer, will always be even. Do you agree? Explain your answer with clear workings.

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.....[3]

- 18** The diagram shows two congruent isosceles triangles, PQS and RSQ .
 PQ and RS are parallel to the x -axis.



- (a)** Write down the coordinates of the point Q .

Answer (..... ,) [1]

- (b)** Find the equation of the line PR .

Answer [2]

- (c)** W is a point on the x -axis such that RW is perpendicular to RS .
 Write down the coordinates of the point W .

Answer (..... ,) [1]

- (d) Find the perimeter of $PQRS$.

Answer units [3]

- 19** In the year 2022, it was recorded that the average amount of waste generated in Singapore daily was 20.3 thousand tonnes. Assuming that there are 365 days in a year, find the total amount of waste generated in the year 2022, leaving your answer in standard form.

Answer tonnes [1]

- 20** A company sells chocolate muffins and banana muffins. These muffins are sold across three different outlets, which are Outlet *A*, Outlet *B* and Outlet *C*. The table shows the number of boxes of chocolate muffins and banana muffins produced at each of the three outlets in a particular day.

	Chocolate Muffin	Banana Muffin
Outlet <i>A</i>	36	40
Outlet <i>B</i>	48	39
Outlet <i>C</i>	45	x

The table can be represented by matrix **Q**, where $\mathbf{Q} = \begin{pmatrix} 36 & 40 \\ 48 & 39 \\ 45 & x \end{pmatrix}$.

- (a) (i) The production cost for each box of chocolate muffin and each box of banana muffin is \$3 and \$4.50 respectively. Represent this information in a 2×1 matrix, **P**.

$$\text{Answer } \mathbf{P} = \begin{pmatrix} & \\ & \end{pmatrix} [1]$$

- (ii) Evaluate the matrix $\mathbf{R} = \mathbf{QP}$.

$$\text{Answer } \mathbf{R} = \begin{pmatrix} & \\ & \end{pmatrix} [1]$$

- (b) State what each of the elements in matrix **R** represents.

.....
 [1]

The company sells each box of muffins for 70% more than it costs to produce.

On that day, the company sold $\frac{5}{6}$ of each flavour of muffins in Outlet A, $\frac{4}{5}$ of each flavour of muffins in Outlet B, and all the muffins in Outlet C.

- (c) By using the matrix **R**, find the total amount collected from the sale of muffins from all three outlets.

Answer \$ [3]

- (d) The total amount collected from the sales of muffins from all three outlets is \$1454.52. Calculate the value of x .

Answer $x =$ [1]

- 21** In a class of 36 students, each student is either in a sports team or a performing arts club. Each student is also either a member or is a leader in their respective co-curricular activities. One student is selected at random from the class. The probability that the student selected is in a sports team is $\frac{1}{3}$.

- (a) Given that there are 4 sports team members in the class, find the number of students who are leaders in a sports team.

Answer [1]

Two students are selected at random from the class. The probability that both of them are members of a performing arts club is $\frac{1}{42}$.

- (b) Find the number of students who are members in a performing arts club.

Answer [3]

In another class, the probability that a student is a leader in a co-curricular activity is k . The probability that a student is in a performing arts club is m .

- (c) Derrick claimed that the probability of a student in this class being a leader or is in a performing arts club is $k + m$. Explain with reasons whether you agree with Derrick's claim.

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.....[1]

- 22** The scale of a map is $1 : n$.
A school garden measuring 8 cm^2 on a map has an actual area of 2048 m^2 .
Find n .

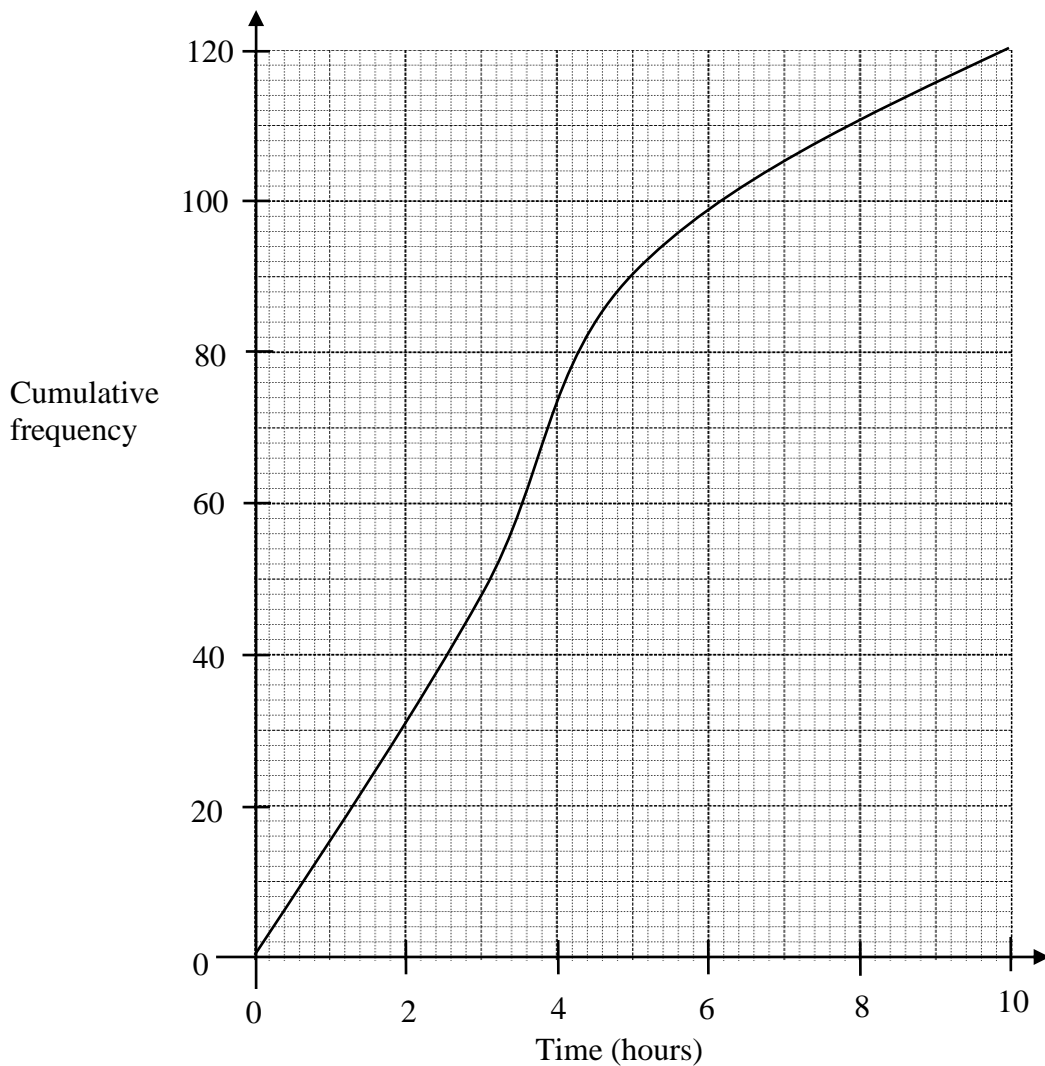
Answer $n = \dots\dots\dots$ [2]

- 23** 4 technicians plan to repair 416 computers in 16 days.
However, after repairing for 10 days, one of the technicians fell sick and could not continue.
The remaining computers are to be repaired by the other three technicians.

Assuming that the technicians work at the same rate, calculate the total number of days needed for all 416 computers to be repaired.

Answer $\dots\dots\dots$ days [3]

- 24 The cumulative frequency diagram shows the time in hours that each of the 120 adults from Town A spend on exercise in one week.



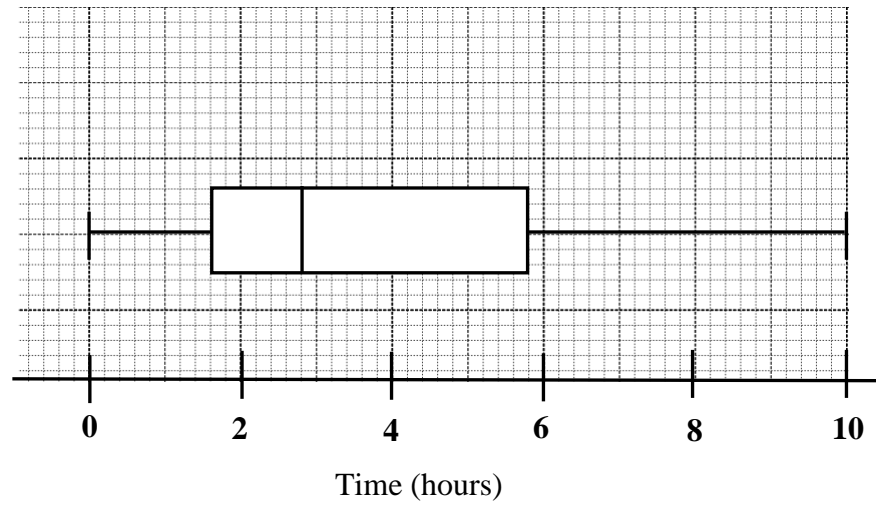
- (a) Explain if you can find the probability of an adult spending exactly 5 hours on exercise weekly.

.....
[1]

- (b) The health promotion board recommends at least k hours of weekly exercise for adults to stay fit and active. 60 % of the adults in Town A meet this recommended weekly exercise hours. Find the value of k .

Answer $k = \dots\dots\dots$ [1]

The box-and-whisker plot shows the time in hours that each of the 120 adults from Town *B* spend on exercise in one week.



- (c) Find the median and interquartile range of the weekly exercise hours of the adults in Town *B*.

Answer Median = h [1]

Interquartile range = h [1]

- (d) Make two comparisons between the weekly exercise hours of the adults from Town *A* and Town *B*. State your reasons.

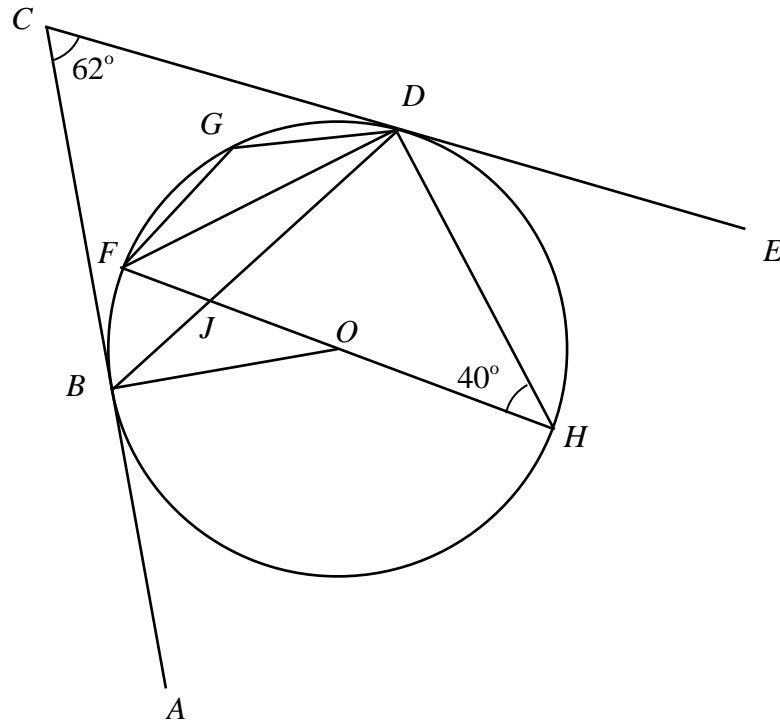
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.....[2]

- 25** The lines ABC and CDE are tangents to the circle at B and D respectively. F , G and H are points on the circle and the centre of the circle is O . The straight lines BD and HF intersect at J . It is given that $\angle ACE = 62^\circ$ and $\angle DHF = 40^\circ$.



Find, giving reasons for each answer,

(a) $\angle FGD$,

Answer $^\circ$ [1]

(b) $\angle CDO$,

Answer $^\circ$ [1]

(c) $\angle BOF$,

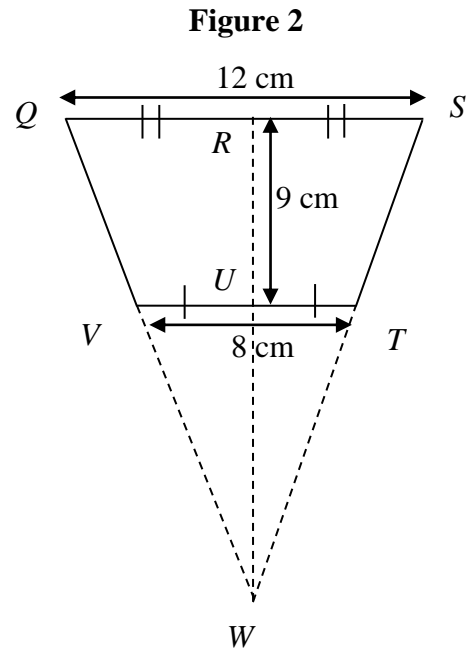
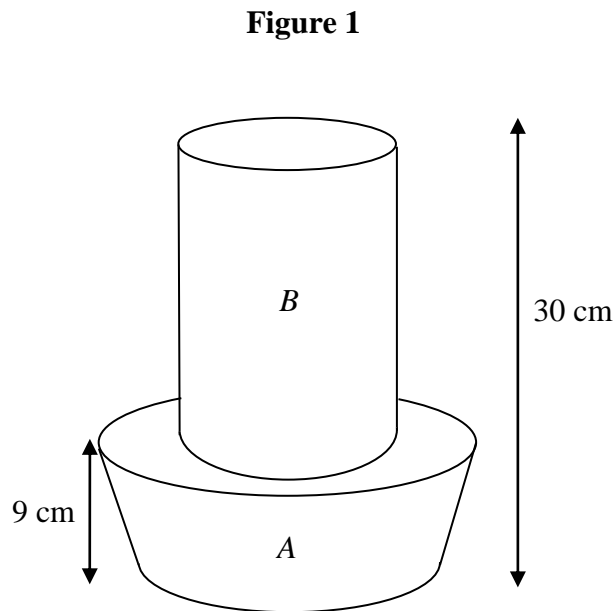
Answer^o [2]

(c) $\angle FJD$.

Answer^o [2]

- 26 A liquid container, 30 cm tall, can be modelled by a frustum, A , placed below a cylinder B , as shown in **Figure 1**.

Figure 2 shows a sketch of the cone that the frustum A is a part of. The diameter QRS of the top of the frustum, is 12 cm and the diameter VUT of the bottom is 8 cm. The height of the frustum, RU is 9 cm. R and U are the midpoints of QS and VT respectively.

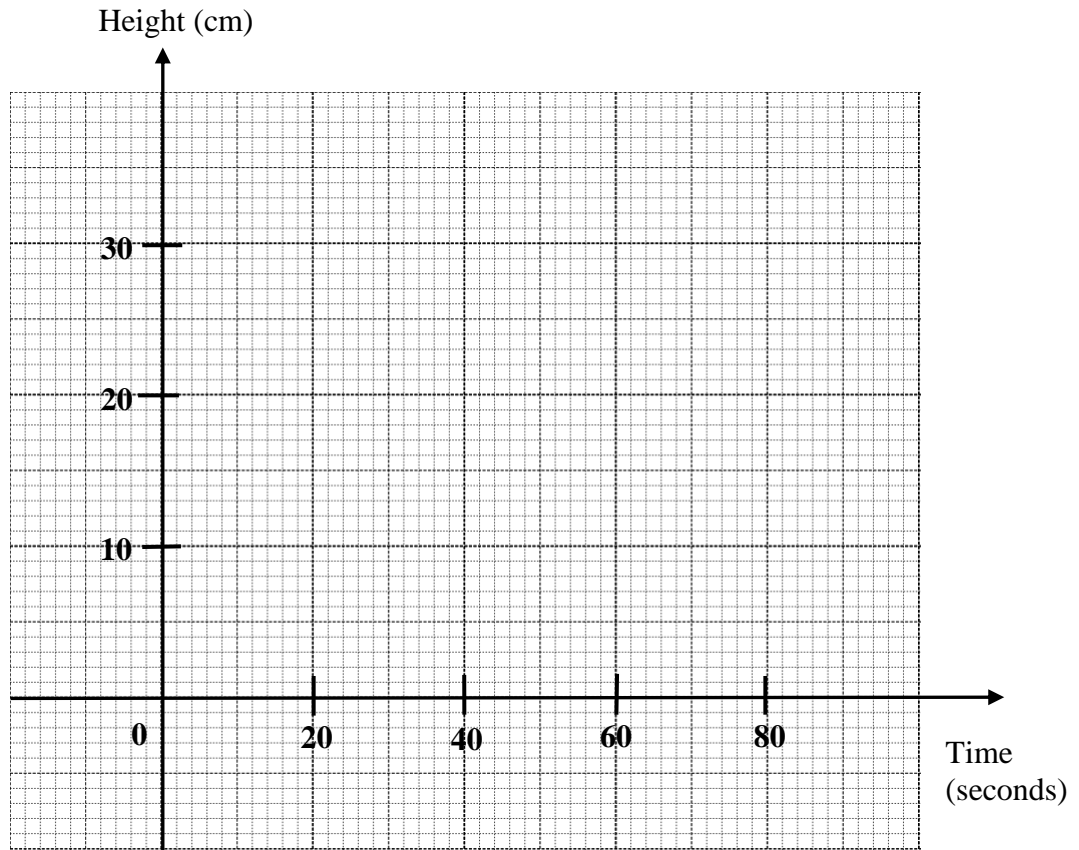


- (a) Calculate the volume of the frustum, A .

Answer cm^3 [3]

A liquid is poured into the container at a constant rate of $q \text{ cm}^3$ per second. It takes 50 seconds to fill up A completely and it takes another 30 seconds to fill up B completely.

(b) Sketch the height of the liquid in the container with respect to time.



[2]

END OF PAPER