Full Name	Class Index No	Class

Anglo-Chinese School
(Barker Road)

PRELIMINARY EXAMINATION 2024
SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)

MATHEMATICS
4052
PAPER 1

2 HOURS 15 MINUTES

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use

This document consists of **21** printed pages and **1** blank page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1   Write the following numbers in order of size, starting with the smallest.

$$\sqrt{0.81}, \quad 0.902, \quad \frac{399}{441}, \quad 0.86^{\frac{2}{3}}$$

Answer _____ , _____ , _____ , _____   [1]

---

2   Write as a single fraction in its simplest form $\dfrac{7x}{(x-5)^2} + \dfrac{1}{5-x}$.

Answer _____   [2]

---

3   The capacity of a SD card is 250 gigabytes correct to 2 significant figures.
The size of a picture file is 2.5 megabytes correct to 1 decimal place.

Calculate the largest number of files that can be stored in this SD card.
Give your answer in standard form, correct to 3 significant figures.
(1 gigabyte = 10^9 bytes, 1 megabyte = 10^6 bytes)

Answer _____   [2]

3

4 Simplify $\dfrac{5c}{2} \div \dfrac{20c^2}{d}$.

Answer _____ [2]

---

5 y is directly proportional to x^2.
If x is increased by 200%, find the percentage increase in y.

Answer _____ % [2]

---

6 A sum of money was divided between Amelia, Brandon and Claire in the ratio 2 : 3 : 4.
If instead, this money had been divided equally between them, Amelia would have received an extra $20.
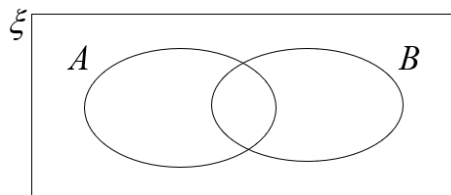What was the total sum of money?

Answer $ _____ [2]

4

7 **(a)** Solve the inequalities $-8 \le 2 - 3x < 7 - \dfrac{1}{2}x$.

Answer _____ [2]

(b) Write down all the integers that satisfy $-8 \le 2 - 3x < 7 - \dfrac{1}{2}x$.
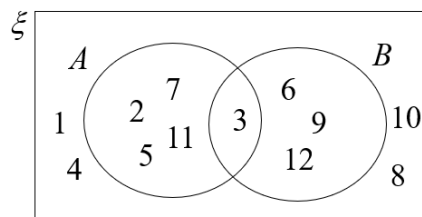
Answer _____ [1]

8 **(a)** On the Venn diagram, shade the region which represents $A \cup B'$.

[1]

(b) $\xi = \{$integers $x : 1 \le x \le 12\}$
$A = \{2, 3, 5, 7, 11\}$
$B = \{3, 6, 9, 12\}$
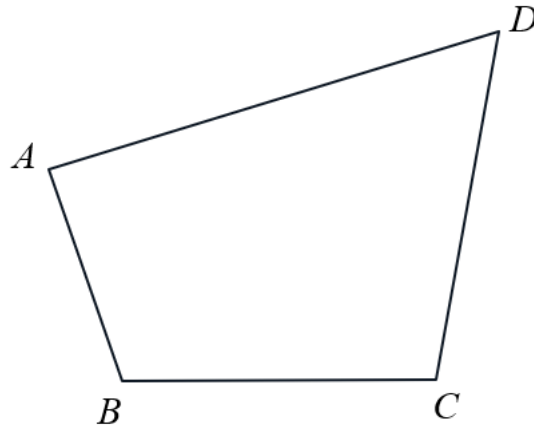
This information is shown on the Venn diagram.

Use one of the symbols below to complete each statement.

$$\varnothing \quad \in \quad \subset \quad \notin \quad \not\subset$$

(i) 7 _____ B [1]

(ii) $\{3, 7\}$ _____ A [1]

9   The diagram represents a plot of land, *ABCD*, which is to be used for a park.

(a)   Construct the bisector of angle *ABC*.   [1]

(b)   Construct the perpendicular bisector of *AD*.   [1]

(c)   A toilet is to be built in the park, nearer to *A* than to *D* and nearer to *BC* than to *AB*.
Shade the region where the toilet is to be built.   [1]

---

10   Using factorisation, solve $6p^2 - 9p - 6 = 0$.

Answer   *p* = _____ or _____   [3]

6

11 A supplier sells watches at $210 each.
Jimmy buys the watches from the supplier at a discount of 20%.
Jimmy intends to then sell the watches at a profit of 20%.

As a marketing strategy, Jimmy plans to offer a 10% discount on the marked price
without affecting his intended 20% profit.
Calculate the marked price that Jimmy should sell each watch at.

Answer $_____ [3]

12 Jim sets off at 08 50 to walk 1000 metres at an average speed of 4 km/h.
He takes a 5 minutes break and then runs 1.3 km in 5 minutes.
   (a) He aims to complete the entire exercise by 09 30. Will he achieve his target?
Show your working clearly.

Answer _____ [2]

_____

   (b) Calculate his average speed for the entire exercise.
Give your answer in metres per minute.

Answer _____ m/min [1]
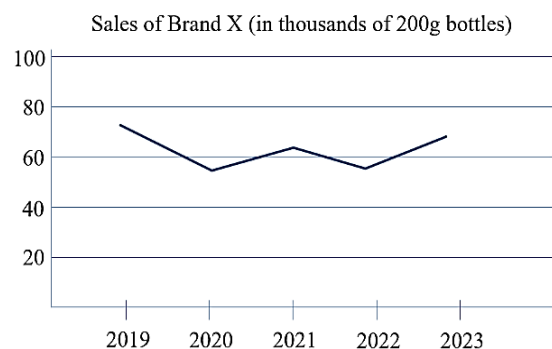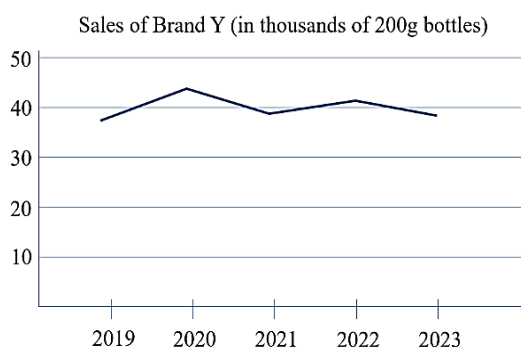
7

13 **(a)**

Brand
A

Brand
A

Two bottles of Brand A coffee are geometrically similar.
The larger bottle contains 200 g of coffee granules.

The larger bottle is approximately 49% taller than the smaller bottle.
Find, in grams, the amount of coffee granules in the smaller bottle.
Give your answer to the nearest gram.

Answer _____ g   [2]

(b)

Sales of Brand Y (in thousands of 200g bottles)

50
40
30
20
10

2019   2020   2021   2022   2023

Sales of Brand X (in thousands of 200g bottles)

100
80
60
40
20

2019   2020   2021   2022   2023

Brand Y claims that its sales of 200 g bottles of coffee are better than that of Brand X
in the past 5 years.
State one aspect of the charts that makes this claim wrong.

Answer _____

_____   [1]

8

14 **(a)** Express 1260 as a product of its prime factors.

Answer _____ [1]

(b) The highest common factor of two numbers is 12 while the lowest common multiple is 1260. Both numbers are less than 200.
Find the two numbers.

Answer _____ and _____ [2]

(c) $486 = 2 \times 3^5$.
m and n are both prime numbers.
Find the values of m and n so that $486 \times \dfrac{m}{n}$ is both a perfect square and a perfect cube.
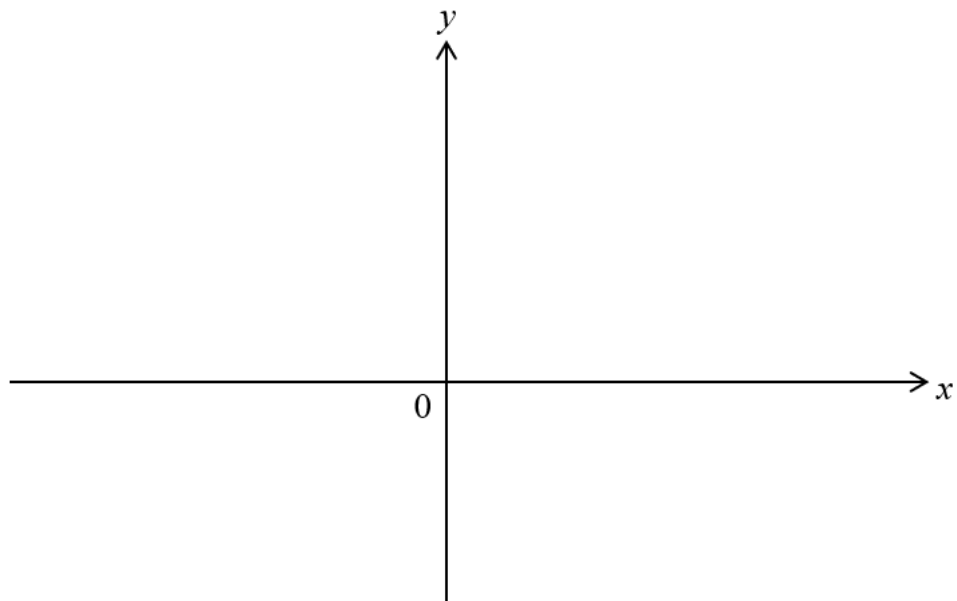
Answer $m =$ _____ and $n =$ _____ [1]

9

15 **(a)** Express $x^2 + \dfrac{1}{2}x + 1$ in the form $(x+a)^2 + b$.

Answer _____ [2]

(b) Sketch the graph of $y = x^2 + \dfrac{1}{2}x + 1$ on the axes below.

Indicate clearly the coordinates of the point where the graph crosses the *y*-axis and the turning point on the curve.

Answer

[2]

10

16 The table below shows the number of cars and motorcycles passing through an Electronic Road Pricing (ERP) gantry on certain days of the week from 7.30 am to 7.55 am.

	Cars	Motorcycles
Wednesday	320	120
Thursday	380	100
Friday	410	130
Charges per vehicle	$2	$0.50

(a) Represent the number of vehicles passing through the ERP gantry in a 3×2 matrix **V**.

$$Answer \quad \mathbf{V} = \begin{pmatrix} & \\ & \\ & \end{pmatrix}$$ [1]

(b) Write down a matrix **C** such that the product $\mathbf{P} = \mathbf{VC}$ represents the total charges incurred by all vehicles on each of the days.

$$Answer \quad \mathbf{C} =$$ [1]

(c) Evaluate the matrix $\mathbf{P} = \mathbf{VC}$.

$$Answer \quad \mathbf{P} =$$ [1]

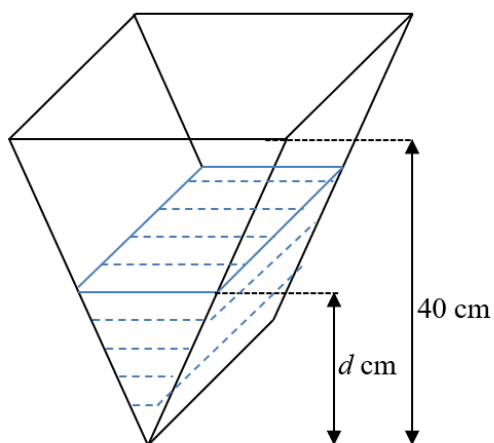**(d)** **A** is a matrix such that $\mathbf{A} = \dfrac{1}{3}\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{P}$.

State what **A** represents.

*Answer*_____

_____ [1]

17

40 cm

d cm

The diagram shows a container in the shape of a prism with a triangular cross-section.
The container has a height of 40 cm.
Water is poured into the empty container at a constant rate.
It takes 12 minutes to fill the container completely.
After t minutes, the depth of the water is d cm.

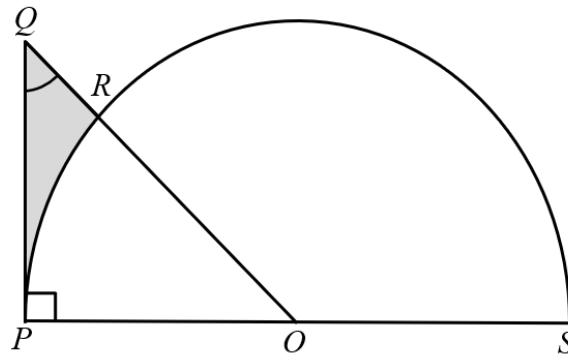**(a)** Find the value of t when $d = 20$.

Answer $t =$ _____ [2]

(b) On the grid, sketch the graph showing how the depth varies during the 12 minutes.

Answer

Depth
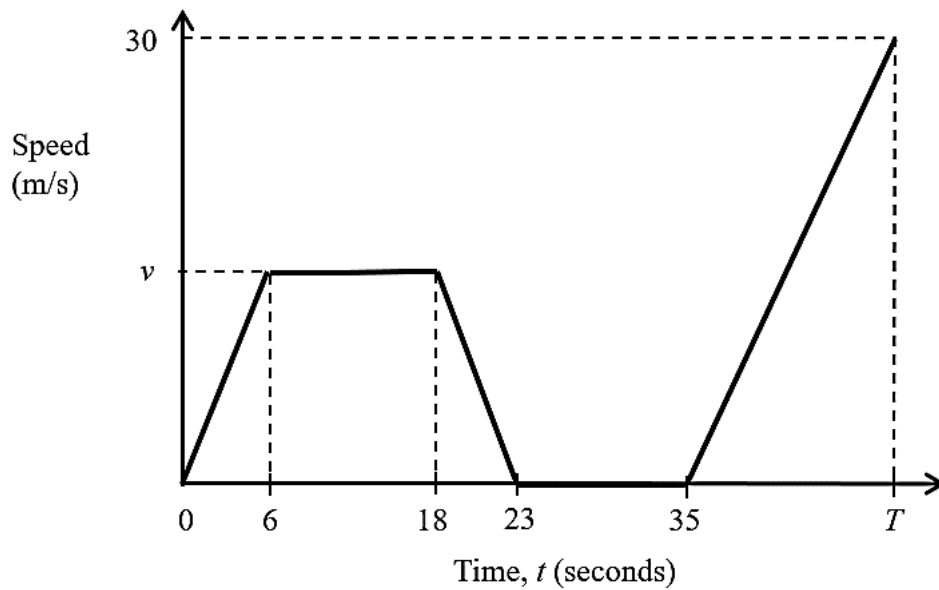(d cm)

Time (t minutes)

[2]
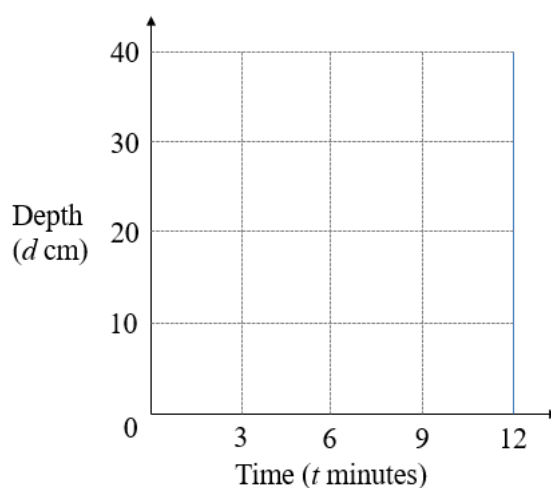
12

18   The diagram shows a semicircle with centre O and radius 8 cm.
   OP is perpendicular to *PQ*.
   Angle *PQO* = 0.7 radians.

Find the area of the shaded region.

Answer _____ cm^2 [4]

19 The diagram below shows the speed-time graph for a car travelling on the road.
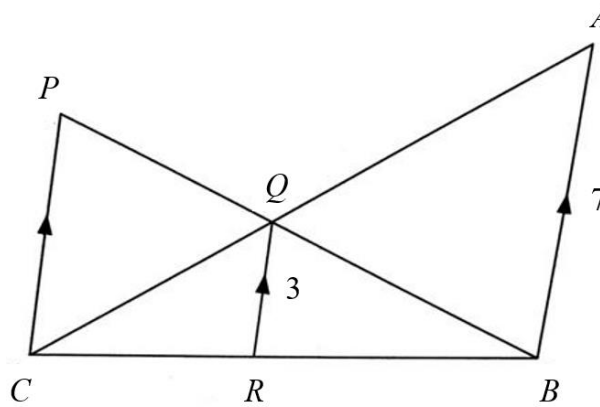

(a) The total distance travelled by the car from 0 to 23 seconds is 385 metres.
Calculate the value of v.

Answer $v =$ _____ [2]

(b) When $t = 45$, the car is travelling at a speed of 25 m/s.
When $t = T$, the car is travelling at a speed of 30 m/s.
Find the value of T.

Answer $T =$ _____ [2]

14

20 In the diagram, *AB*, *QR* and *PC* are parallel.
PQB, *CQA* and *CRB* are straight lines.
AB = 7 cm and *QR* = 3 cm.

(a) Show that triangle *PCB* and triangle *QRB* are similar.
Give a reason for each statement.

Answer

_____

_____

_____

_____ [2]

(b) Write down another pair of similar triangles.

Answer Triangles _____ and _____ [1]

(c) Calculate *PC*.

Answer _____ cm [2]

---

21 **(a)** Simplify $\left(\dfrac{4a^6}{b^4}\right)^{-\frac{1}{2}}$, giving your answer in positive index.

Answer _____ [2]

(b) $\dfrac{2^k}{\sqrt[4]{8}} = 4^{2k}$.

Use the laws of indices to find the value of k.
Show your working.

Answer $k =$ _____ [3]

16

22 The first three terms in a sequence of numbers are given below.

$$T_1 = 1 \times 2 + 10 = 12$$
$$T_2 = 2 \times 3 + 6 = 12$$
$$T_3 = 3 \times 4 + 2 = 14$$
$$T_4 = 4 \times 5 - 2 = 18$$

(a) Show that the nth term of the sequence, T_n, is given by $T_n = n^2 - 3n + 14$.
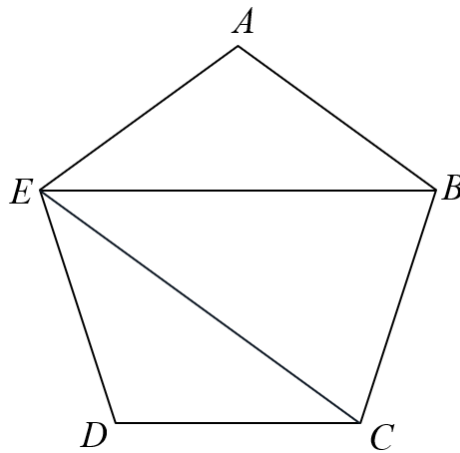
Answer

[2]

(b) Evaluate T_{50}.

Answer _____ [1]

(c) Explain why every term in the sequence is even.

Answer _____

_____

_____

_____ [2]

17

23 In the diagram, *ABCDE* is a regular pentagon.

(a) Show that *BE* is parallel to *CD*.

Answer _____

_____

_____

_____

_____

_____

_____ [3]

(b) The sides *AB* and *DC* are produced to meet at *X*.
What type of quadrilateral is *BECX*?
Explain your answer.

Answer _____

_____

_____

_____

_____

_____

_____ [3]

18

24 *A* is the point (8, –2) and $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

    (a) Find

        (i) $\left| \overrightarrow{AB} \right|$,

Answer _____ [1]

        (ii) the equation of the line *AB*.

Answer _____ [2]

    (b) The equation of the line *CD* is $2y - 4x = 19$.
        Find the coordinates of the point of intersection of *AB* and *CD*.

Answer (_____ , _____) [3]

19

25 There are 12 boys and 13 girls in a class.
Two of the children are selected at random to participate in a forum.
    (a)   Draw a tree diagram below to show the probabilities of the possible outcomes.

[2]

    (b)   Find, as a fraction in its simplest form, the probability that
        (i)   two boys are selected,

Answer _____ [1]

        (ii)   one boy and one girl are selected.

Answer _____ [2]

20

(c) As a standby, a third student is selected.
Find the probability that out of the three students, there will be at least one boy and one girl selected.

Answer _____ [2]

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