

# TAMPINES SECONDARY SCHOOL

Secondary Four Express / Five Normal Academic  
Preliminary Examination 2024

NAME

CLASS

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REGISTER  
NUMBER

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**ADDITIONAL MATHEMATICS**

**4049/01**

**Paper 1**

**28 August 2024**

**2 hours 15 minutes**

Candidates answer on the Question Paper.

## READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.  
Write in dark blue or black pen.  
You may use an HB pencil for any diagrams or graphs.  
Do not use staples, paper clips, glue or correction fluid.

**For Examiner's Use**

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.  
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

## *Mathematical Formulae*

### 1. ALGEBRA

#### *Quadratic Equation*

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#### *Binomial expansion*

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$ .

### 2. TRIGONOMETRY

#### *Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

#### *Formulae for $\triangle ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1 The equation of a curve is  $y = \frac{2x^2}{x+1}$  where  $x \neq -1$ . Determine the range of values of  $x$  for which  $y$  is a decreasing function. [4]

- 2 A missile, TP-1 was launched such that its height,  $h_1$  metres above the ground was given by  $h_1(x) = -20x^2 + 120x + 3$ , where  $x$  metres was the horizontal distance of the missile from the launched position.

(a) Express  $-20x^2 + 120x + 3$  in the form  $a(x + b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are constants. [2]

Another missile, TP-2 was launched from the same position as TP-1. The height of TP-2,  $h_2$  metres above the ground was given by  $h_2(x) = -\frac{49}{10}(x - 6)^2 + 183$ .

(b) Find  $h_1(0)$  and  $h_2(0)$  and hence interpret the meaning of your answers. [2]

(c) The missile that could reach a greater height and travel a distance further away from its launched position was acquired. Determine if missile TP-1 or TP-2 was acquired. Show your working clearly. [3]

- 3 A cuboid with volume  $(2x + 1)^2 \text{ cm}^3$  has a rectangular base with dimensions  $x^2 \text{ cm}$  and  $(2x - 1) \text{ cm}$ . Find an expression for the height of the cuboid, leaving your answer in partial fractions. [6]

4 (a) (i) Find in ascending powers of  $x$ , the simplified first three terms in the expansion of  $(2 + qx)^6$ . [3]

(ii) Given that the first two non-zero terms in the expansion of  $(2 + px)(2 + qx)^6$ , where  $q > 0$ , are 128 and  $-168x^2$ , find the values of  $p$  and  $q$ . [4]

- (b) Find the term independent of  $x$  in  $\left(x^3 - \frac{2}{x}\right)^{12}$ . [3]

- 5** A function  $f(x)$  is defined for all real values of  $x$  such that  $f''(x) = 18e^{-3x}$ . The gradient of the curve  $y = f(x)$  at  $x = 0$  is 2 and the curve passes through  $\left(1, \frac{2}{e^3}\right)$ .

**(a)** Find the exact value of the  $x$ -coordinate of the stationary point of the curve and determine its nature.

[6]

**(b)** Find the equation of the curve.

[2]



**6** A graph has the equation  $y = x^2 + (6 - 2m)x + m + 5$  where  $m$  is a constant.

**(a)** Given that the graph cuts the  $x$ -axis at  $A$  and  $B$  and the point  $(4, 0)$  is the midpoint of  $AB$ , find the value of  $m$ . [3]

**(b)** If  $m = 8$ , find the range of values of  $p$  such that the graph of  $y = x^2 + (6 - 2m)x + m + 5 + p$  lies above the  $x$ -axis. [2]

- 7 The number of insects present in a colony  $t$  weeks after observation began, can be modelled by the equation  $m = ab^{\frac{t}{3}}$ . Measurements of  $m$  and  $t$  are shown in the table below.

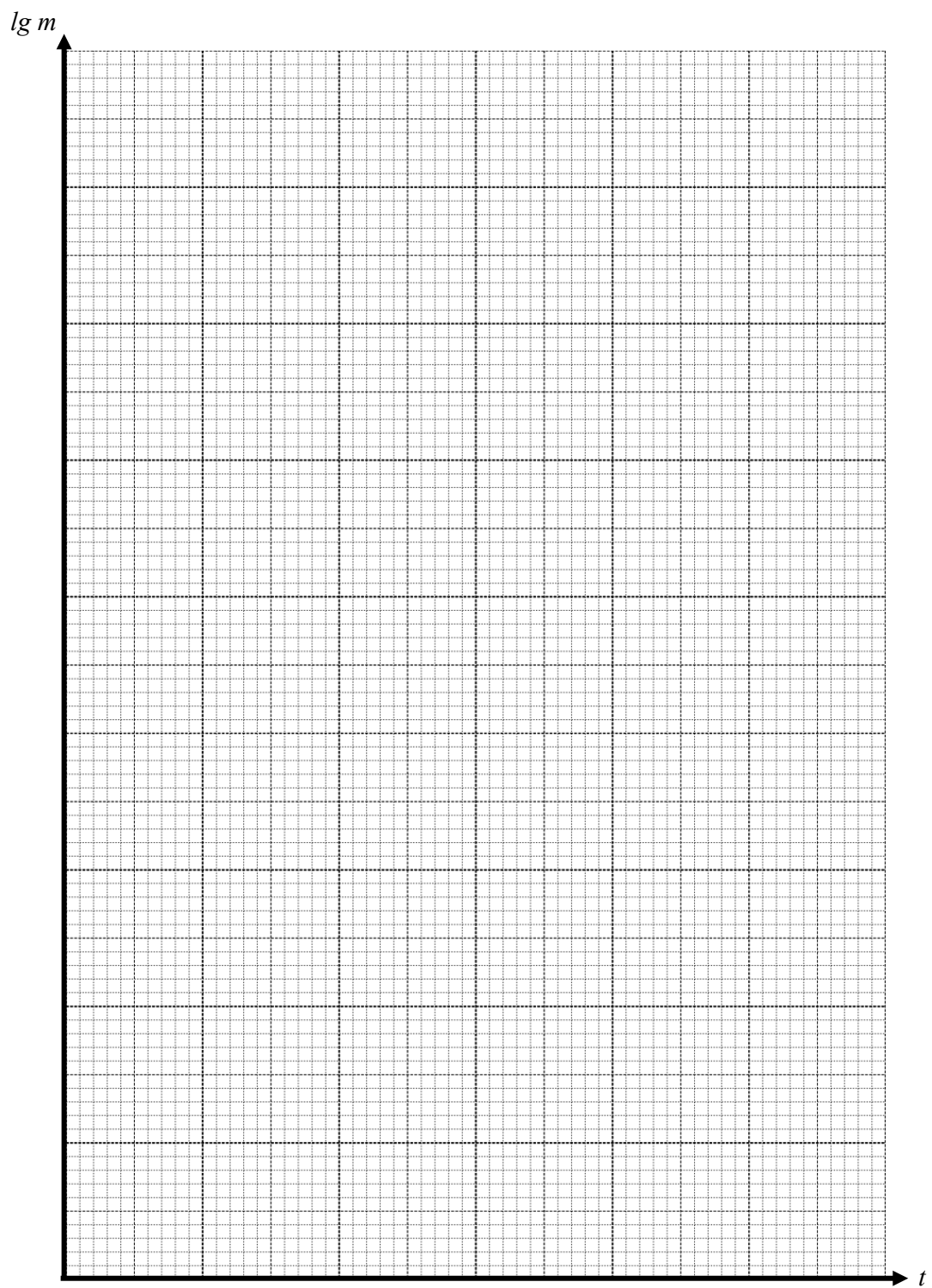
$t$	2	4	6	8	10
$m$	920	1108	1333	1605	1930

- (a) Plot  $\lg m$  against  $t$  and draw a straight line graph to illustrate the information. [2]

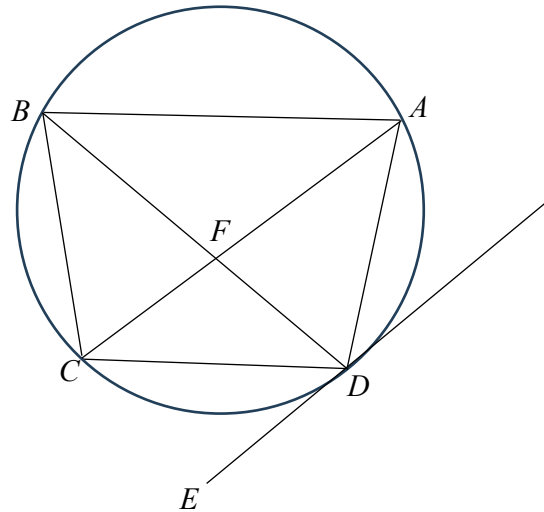
- (b) Use your graph to estimate the values of  $a$  and  $b$ . [3]

The number of insects present in another colony  $t$  hours after observation began, can be modelled by the equation  $\lg m^{40} = t + 120$ .

- (c) By drawing a suitable straight line on your graph estimate the time taken for the two colonies to have the same number of insects. [2]



8



In the diagram,  $A$ ,  $B$ ,  $C$  and  $D$  lie on the circumference of the circle such that  $BD$  is the diameter of the circle.  $BD$  and  $AC$  intersect at  $F$ .  $DE$  is a tangent to the circle at  $D$  and  $AD = CD$ .

**(a)** Show that  $AC$  is parallel to  $DE$ .

[3]

**(b)** Prove that triangle  $ABD$  is similar to triangle  $CBD$ .

[3]

- 9 The height,  $y$  metres of the water level near a beach can be modelled by the equation  $y = a - b \cos(pt)$ , where  $t$  is the number of hours after midnight. and  $p$  is the radians per hour.

A low tide is observed at midnight and the duration between successive low tides is 12 hours.

- (a) Show that  $p = \frac{\pi}{6}$ . [1]

The greatest and least heights of the water level are 8 metres and 4 metres respectively.

- (b) State the value of  $a$  and of  $b$  for  $a > 0$  and  $b > 0$ . [2]

(c) Hence sketch the graph of  $y = a - b\cos(pt)$  for  $0 \leq t \leq 12$ .

[3]



(d) People have been advised to stay away from the beach when the height of the water level is 6 metres or higher. Determine the periods from midnight to 23 00 when the people must stay away.

[2]

- 10** A particle is at 3 metres past a fixed point  $O$ . It starts to move in a straight line such that  $t$  seconds later, its velocity  $v$  m/s is given by  $v = t^2 - 6t + 5$ . The particle comes to an instantaneous rest first at point  $A$  then at point  $B$ .

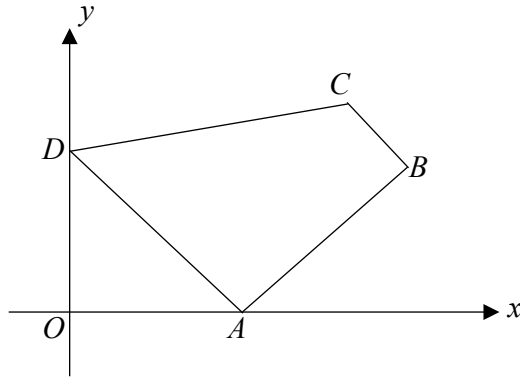
**(a)** Find an expression, in terms of  $t$ , for the distance of the particle from  $O$  at  $t$  seconds. [2]

**(b)** Find the total distance travelled by the particle in the first 5 seconds. [5]



- (c) Point  $C$  is where the particle has zero acceleration. Determine if point  $C$  is nearer to its initial starting position or point  $B$ . [3]

- 11** The trapezium  $ABCD$  is such that  $AD$  is parallel to  $BC$  and  $A$  is  $(2, 0)$ . The equation of  $BC$  is  $y = 11 - 3x$  and  $2BC = AD$ .



- (a)** Find the equation of  $AD$ .

[2]

- (b)** Find the equation of the perpendicular bisector of  $AD$ .

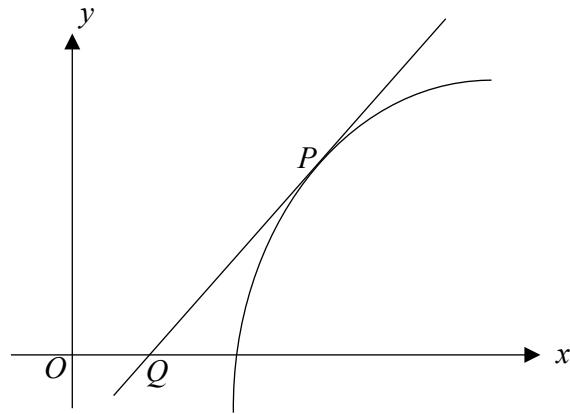
[3]

(c) The perpendicular bisector found in part (b) passes through the point  $C$ . Find the coordinates of  $C$ . [2]

(d) Show that  $B$  has coordinates  $\left(\frac{7}{2}, \frac{1}{2}\right)$ . [1]

(e) Hence find the area of trapezium  $ABCD$ . [2]

12



The diagram shows part of the curve  $y = 2 - \frac{3}{4x-5}$  for  $x > \frac{5}{4}$ .

(a) Determine if the curve has a stationary point.

[3]

The tangent to the curve at  $P$  cuts the  $x$ -axis at  $Q\left(\frac{5}{4}, 0\right)$ . The normal to the curve at  $P$  is parallel to

$$y = -\frac{3}{4}x + 10.$$

**(b)** Find the coordinates of  $P$ .

[3]

**(c)** The normal to the curve at  $P$  cuts the  $x$ -axis at  $R$ . Find the area of triangle  $PQR$ .

[3]

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