



ST. MARGARET'S SCHOOL (SECONDARY)

Preliminary Examinations 2024

CANDIDATE NAME

CLASS

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REGISTER NUMBER

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ADDITIONAL MATHEMATICS

4049/02

Paper 2

20 August 2024

Secondary 4 Express / 5 Normal (Academic)

2 hours 15 minutes

Candidates answer on the Question Paper.

Additional Materials: NIL

READ THESE INSTRUCTIONS FIRST

Write your name, registration number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place. For π , use either your calculator or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 90.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \dots (n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1** The growth in population of bacteria in an experiment is modelled by the equation $P = 80e^{mt}$, where P is the population of the bacteria t hours after the start of the experiment.
- (a)** Find the initial number of bacteria when the researcher started the experiment. [1]
- (b)** The population of the bacteria grew to 1600 after 12 hours. Show that $m = 0.24964$, correct to 5 significant figures. [2]
- (c)** Find the amount of bacteria after 1 day. Round off answer to the nearest whole number. [2]
- (d)** The experiment will end when the bacteria population reaches 1.5 million. A researcher claims that he will complete his experiment in 2 days. Is his claim true? Show your working clearly. [3]

- 2** It is given that $f(x) = 3x^3 + ax^2 + bx - 18$, where a and b are constants. $x - 2$ is a factor and leaves a remainder of -51 when divided by $x + 1$.

(a) Find the values of a and b ,

[4]

(b) Explain why $f(x) = 0$ has only one real root.

[3]

3 A calculator must not be used in this question.

(a) Use the identity for $\cos 2A$ to show that $\sin^2 15^\circ = \frac{2 - \sqrt{3}}{4}$. [3]

(b) Prove that $\sin^2 15^\circ + \sin^2 75^\circ = 1$. [5]

- 4 (a) Prove the identity $\frac{1}{2 \tan \theta + 2 \cot \theta} = \frac{1}{4} \sin 2\theta$. [4]

- (b) Hence solve the equation $\frac{1}{2 \tan \theta + 2 \cot \theta} = \sin \theta$, for $0 \leq \theta \leq 2\pi$. Explain why [3]
there is no real solution for equation.

- 5 **(a)** Show that the solution of the equation $6^x \times 4^{x-1} = 3^{x+2}$ is $x = \frac{\lg 36}{\lg 8}$. [3]

- (b)** Explain why there is no real solution for the equation $\log_2(4x-3) - \log_2(2x-1) = 2$. [3]

(c) Solve the equation $\log_8 \frac{1}{2} \sqrt{x} = 2 \log_x 16 + 1$.

[5]

6 The equation of a curve is $y = \frac{5x}{\sqrt{2x-3}}$, $x \neq a$.

(a) State the value of a . [1]

(b) Find $\frac{dy}{dx}$. [2]

(c) Hence, evaluate $\int_3^4 \frac{2(x-3)}{(2x-3)^{\frac{3}{2}}} dx$. [3]

- (d) A particle moves along the curve in such a way that the x -coordinate is increasing at a constant rate of 0.2 units per second. Find the rate of change of the y -coordinate at a point where $x = 5$. [2]

- 7 **(a)** Given that $P = 18 \sin \theta \cos \theta + 10 \cos^2 \theta - 3$, show that [2]
 $P = A \sin 2\theta + B \cos 2\theta + C$, where A , B and C are constants.

- (b)** Express P as $R \sin(2\theta + \alpha) + C$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

- (c) State the maximum value of P and the value of the acute angle θ , when P is maximum. [3]

- (d) P cannot be equal to -13 . Is this statement true? Justify your answer without solving for θ . [2]

8 The equation of a circle C_1 is $x^2 + y^2 - 4x - 8y + 4 = 0$, with centre A .

(a) Find the radius and the coordinates of the centre of the circle.

[3]

(b) Explain why the x -axis is a tangent to the circle at the point $P(2,0)$.

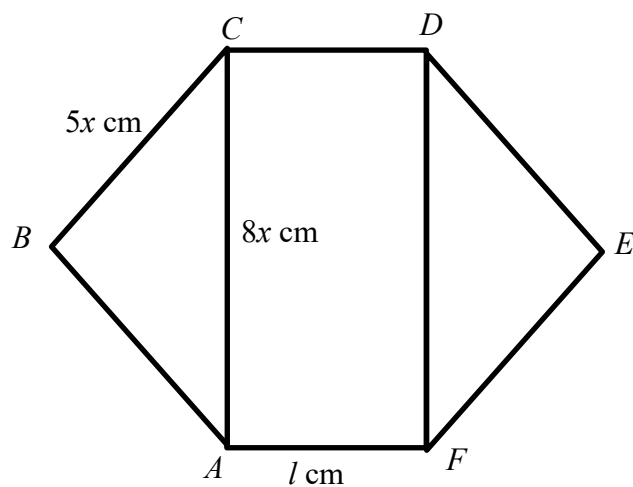
[2]

- (c) Find the exact value of the coordinates of the points at which the circle intersects the y -axis. [3]

- (d) Given that the line joining the centre of the circle and the origin makes an angle θ with the positive x -axis, find the value of θ . [2]

- (e) State the equation of a circle C_2 which is a reflection of C_1 on the line $y = 8$. [2]

- 9 The diagram below shows the cross-sectional area of a stool, which is made up of 2 isosceles triangles and a rectangle. The sides of the isosceles triangle are $5x$ cm each and the length and width are $8x$ cm and l cm respectively.



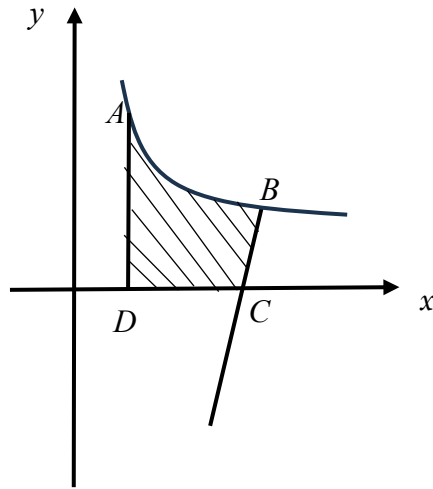
- (a) Given that the perimeter of this cross-section is 148 cm, express l in terms of x . [2]

- (b) Show that the volume of the stool, V , can be represented by the formula $V = 5920x^2 - 560x^3$, if the height of the stool is $10x$ cm. [4]

(c) Given that x can vary, find the value of x for which the volume is stationary. [2]

(d) Determine whether this value of x gives a maximum or minimum value for volume. [2]

- 10 The diagram shows part of a curve $y = \frac{5}{x+1}$, where A and B are points on the curve.



- (a) The equation of line AD is $x = 1$. State the coordinates of A .
Given also that B is the point of intersection between the curve and the line $y = -x + 5$, find the coordinates of B .

[3]

- (b) The normal to the curve at B meets the x -axis at point C . Find the equation of BC . Hence find the coordinates C . [3]

- (c) Find the area of the shaded region. [3]

