



ST. MARGARET'S SCHOOL (SECONDARY)

Preliminary Examinations 2024

CANDIDATE NAME

CLASS

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REGISTER NUMBER

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ADDITIONAL MATHEMATICS

4049/01

Paper 1

15 August 2024

Secondary 4 Express / 5 Normal (Academic)

2 hours 15 minutes

Candidates answer on the Question Paper.

Additional Materials: NIL

READ THESE INSTRUCTIONS FIRST

Write your name, registration number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place. For π , use either your calculator or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 90.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and
$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1) \dots (n-r+1)}{r!}$$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC ,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The equation of a curve is $y = -2x^2 - 12x + 1$.

- (a)** Express $-2x^2 - 12x + 1$ in the form $a(x + b)^2 + c$. Hence state the maximum value of y and its corresponding value of x . [4]

- (b)** The line $y = -2x + 1$ intersects the curve at points A and B .

Find the value of k for which the distance AB can be expressed as \sqrt{k} . [4]

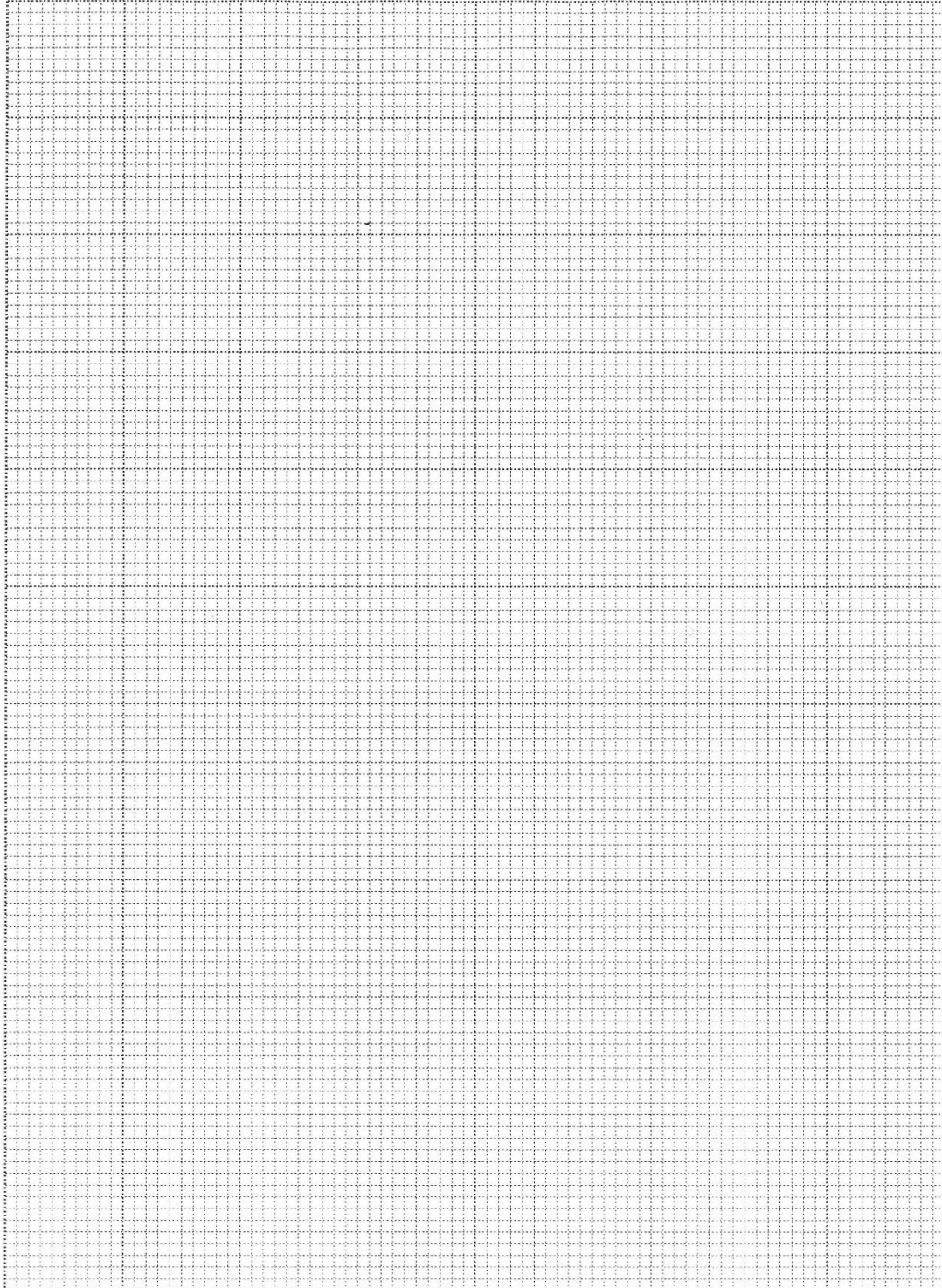
- 2 A pot of cocoa butter is cooled from its initial temperature to a temperature of T °C in x minutes is given by $T = A(B^x)$, where A and B are constants.

The freezing point of the cocoa butter is 16 °C. The table below shows the corresponding values of T and x recorded.

x	5	10	15	20	25
T	35.4	20.9	12.4	7.3	4.3

- (a) Plot $\lg T$ against x and draw a straight line graph to illustrate the information.

[2]



2 (b) Use your graph to estimate the value of each of constants A and B . [4]

(c) Use your graph to explain whether the cocoa butter is frozen at 13 minutes. [2]

- 3 (a)** Express $\frac{3x^2 - 1}{x^2(2x - 1)}$ in partial fractions. [5]

- (b)** Hence, integrate $\frac{3x^2 - 1}{x^2(2x - 1)}$ with respect to x . [3]

4 The equation of a curve is $y = (k - 1)x^2 + 4x + k + 2$, where k is a constant.

(a) Find the range of values of k given that the curve lies completely below x axis. [5]

(b) Find the values of k for which the line $y = -2x + 3$ is a tangent to the curve. [3]

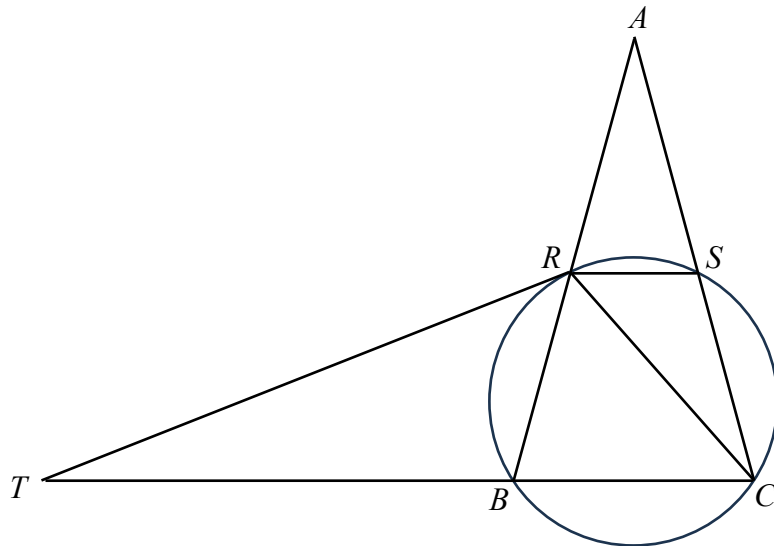
- 5 (a)** By considering the general term in the binomial expansion of $\left(3x - \frac{2}{x}\right)^8$,
explain why there are no odd powers of x in this expansion. [2]

- (b)** Given that there is no term in x^4 in the expansion of $(ax^2 - 1)\left(3x - \frac{2}{x}\right)^8$,
find the value of constant a . [4]

6 A curve is such that $\frac{d^2y}{dx^2} = \sin x + 2 \cos^2 x$. The curve passes through point

$A \left(\frac{\pi}{2}, -\frac{3}{4} \right)$ and the gradient of the curve at A is $\frac{\pi}{2}$. Find the equation of the curve. [7]

7



In the diagram above, R and S are midpoints of AB and AC respectively.

B , C , R and S lie on the circumference of the circle.

TR is a tangent to the circle at R .

TBC is a straight line.

(a) Prove that $\text{angle } SRC = \text{angle } RCB$.

[2]

(b) Prove that triangle TBR is similar to triangle TRC .

[2]

(c) Show that $TR^2 - TB^2 = 2TB \times RS$.

[3]

8 Given that $f(x) = \frac{x^2 - x + 1}{3x - 3}$, $x \neq 1$,

(a) Show that $f'(x) = \frac{x^2 - 2x}{3(x-1)^2}$. [3]

(b) Find the range of values of x for which $f'(x)$ is decreasing. [3]

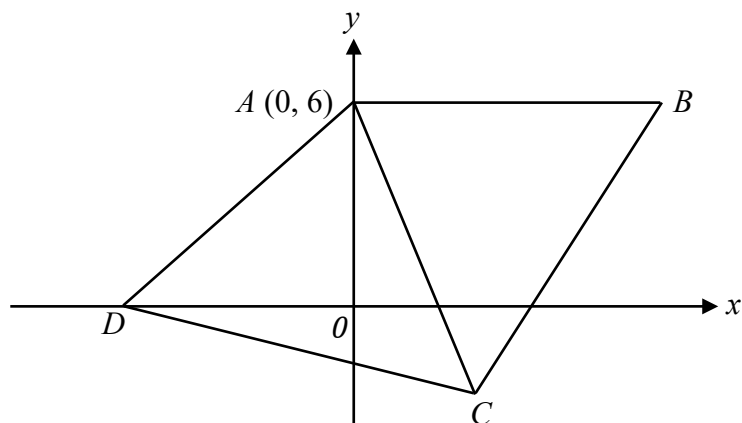
9 (a) (i) State the amplitude of $y = -2 \sin x$. [1]

(ii) State the period of $y = 2 \cos 2x - 1$. [1]

(b) Sketch on the same diagram, the curve $y = -2 \sin x$ and $y = 2 \cos 2x - 1$ for $0 \leq x \leq 2\pi$. [3]

(c) State the number of solutions that satisfy the equation $2 \cos 2x = 1 - 2 \sin x$ for $0 \leq x \leq 2\pi$. [1]

- 10** The diagram shows a quadrilateral $ABCD$ in which A is $(0, 6)$ and AB is parallel to the x -axis. D is a point on the x -axis such that the equation of DC is $x + 5y = -6$. AC is perpendicular to the line $2y - x = 7$.



(a) Find,

(i) the equation of AC ,

[2]

(ii) the coordinates of C .

[2]

- 10 (b)** Given that the area of $\triangle ACD$ is 1.5 times that of $\triangle ABC$, find the coordinates of B . [3]

- 11** The velocity, $v \text{ ms}^{-1}$, of a particle moving in a straight line is given by $v = 6t^2 + kt + 12$, where t is the time in seconds after the particle passes through a fixed point O and k is a constant. Given that, when $t = 1$, the acceleration of the particle is -6 ms^{-2} ,

(a) Show that $k = -18$. [2]

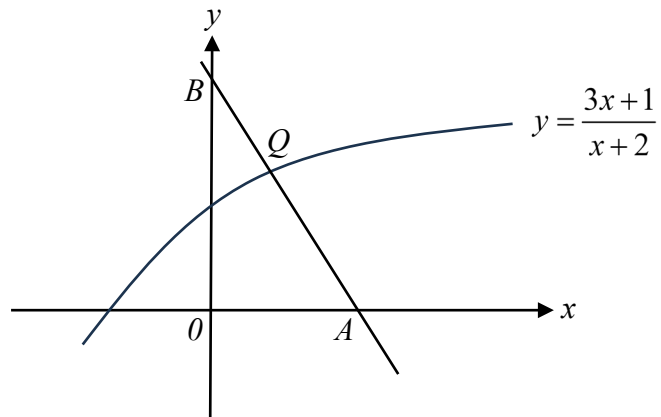
(b) Hence find

(i) the minimum velocity achieved by the particle, [2]

11 (b) (ii) the total distance travelled during the first 4 seconds.

[5]

12



The diagram shows part of the curve $y = \frac{3x+1}{x+2}$ for $x > -2$.

(a) Explain why the curve does not have a stationary point.

[3]

- (b) The point Q lies on the curve and the gradient of the curve at Q is $\frac{1}{5}$. The normal to the curve at Q meets the x -axis at A and the y -axis at B . Find the area of the triangle AOB . [5]

- (c) The line $y = c$ does not intersect the curve. By expressing $\frac{3x+1}{x+2}$ in the form $a + \frac{b}{x+2}$, where a , b and c are constants, explain why $c \geq 3$. [2]

~ End of Paper ~

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