SWISS COTTAGE SECONDARY SCHOOL
SECONDARY FOUR AND FIVE
PRELIMINARY EXAMINATION

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Name: ________________________________ (   )   Class: _________

ADDITIONAL MATHEMATICS

Paper 1

4049/01
Monday 9 September 2024
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

Questions	1	2	3	4
Marks				

This document consists of **18** printed pages and **2** blank pages.

Setter: Mr Heng Teng Boon
Vetter: Mdm Zoe Pow

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Home of Thoughtful Leaders: Serve with Honour, Lead with Humility

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots.. + \binom{n}{r} a^{n-r} b^r + \ldots. + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots\ldots..(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

Section A (17 marks)

1 The equation of a curve is $y = 3x^3 + ax^2 + b$, where a and b are constants. If $a > 0$, find, in terms of a and/or b, the range of values of x for which y is increasing. [3]

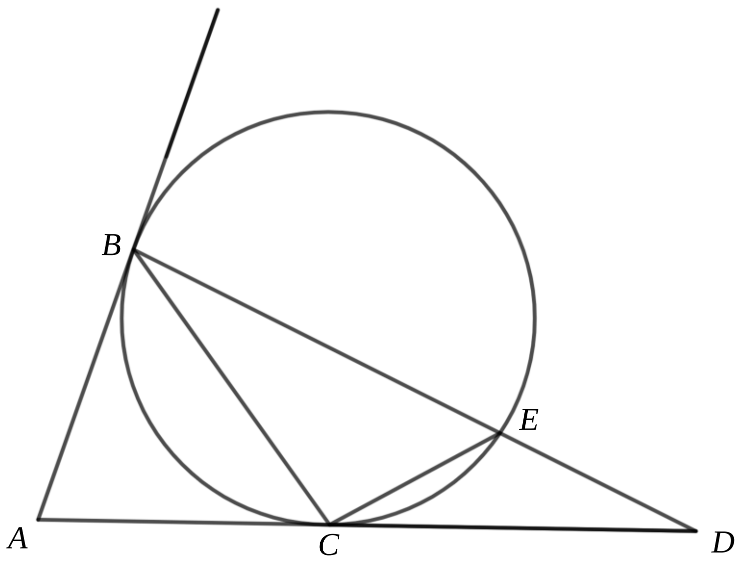
2 The area of a rectangle is $\left(7 + b\sqrt{2}\right)$ cm^2. Given that the length of the rectangle is $\left(a + 4\sqrt{2}\right)$ cm and the breadth of the rectangle is $\left(5 - \sqrt{2}\right)$ cm, find the value of a and of b. [4]

3 The equation of a curve is $y = 2x^2 + 12x + 11$.

(a) Express $2x^2 + 12x + 11$ in the form $a(x+b)^2 + c$ where a, b and c are constants. [2]

(b) Find the range of values of p for which the line $y = px + 11$ intersects the curve at two distinct points. [3]

4



The diagram shows a triangle BCE whose vertices lie on the circumference of a circle. AD is a tangent to the circle at point C and AB is a tangent to the circle at point B. BED is a straight line.

(a) Prove that angle $ABC +$ angle $CED = 180^{\circ}$. [3]

(b) Show that there **does not** exist a circle that passes through points A, B, E and C. [2]

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Name: ____ (   )

Class: ___________

Questions	5	6	7	8	9	10	11	12	13
Marks									

Section B (73 marks)

5 **(a)** Solve the equation $\log_5 x + 2 = 3\log_x 5$. [5]

(b) Sketch the graph $y = \log_{0.5} x$. [2]

6 It is given that $\mathrm{f}(x) = 3\sin\left(\frac{x}{2}\right) + 4$.

(a) State the least and greatest value of $\mathrm{f}(x)$. [2]

(b) State the period of $\mathrm{f}(x)$. [1]

(c) Sketch the graph of $y = \mathrm{f}(x)$ for $0 \le x \le 4\pi$. [2]

(d) By drawing a suitable straight line on the same set of axes as the graph of $y = \mathrm{f}(x)$, state the number of solutions of the equation $\sin\left(\frac{x}{2}\right) = -\frac{x}{4\pi}$ for $0 \le x \le 4\pi$. [2]

7 A curve is such that $\frac{d^2y}{dx^2} = 3e^{-2x} + \cos 2x$. The curve passes through the point A (0, 3) and has a gradient of 5 at A. Find the equation of the curve. [7]

8 **(a)** The expansion of $\left(3x - \dfrac{2}{x^2}\right)^n$ has a term independent of x. By considering the general term in the expansion, explain why n is a multiple of 3. [3]

(b) It is given that $n = 9$.

Find the value of $\dfrac{\text{coefficient of term independent of } x}{\text{coefficient of } \dfrac{1}{x^6}}$. [4]

9 **(a)** Express $\dfrac{9x^2-4x+8}{(x-2)(x+1)^2}$ in partial fractions. [5]

(b) Hence, find $\displaystyle\int \frac{9x^2-4x+8}{(x-2)(x+1)^2}\,\mathrm{d}x$. [3]

10 A particle moves in a straight line such that its displacement, s cm, from a fixed point O is modelled by $s = -3t + e^{\frac{t}{2}}$, where t is the time in seconds since the start of motion.

(a) Show that the particle reaches instantaneous rest at $t = 2\ln 6$. [3]

(b) Explain why the particle passes through O during the first second. [2]

(c) Find the total distance travelled by the particle in the interval $t = 0$ to $t = 4$. [3]

11 The table shows, to 3 significant figures, the value, \$$C$, in thousands, of a car t years from 1^{st} January 2024.

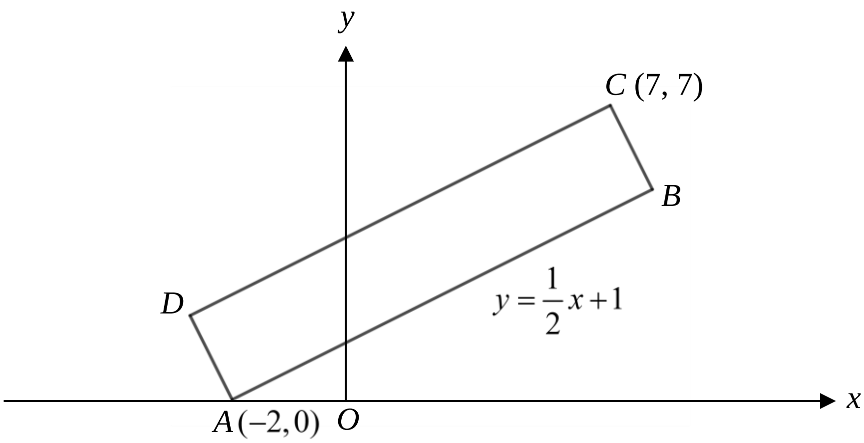
t	1	2	3	4
C	68.2	58.6	50.7	44.3

(a) On the grid below plot $\ln(C-15)$ against t and draw a straight line graph. The vertical $\ln(C-15)$ -axis should start at 3.0 and have a scale of 2 cm to 0.2 units. The horizontal t-axis should have a scale of 2 cm to 1 unit. [3]

(b) Use your graph to find the gradient of your straight line and hence express C in the form $C = Ae^{-kt} + 15$, where A and k are constants. [4]

(c) Assuming that the model is still appropriate, find the year for which the value of the car is first below \$35 000. [3]

12



The diagram shows a parallelogram with vertices $A(-2, 0)$, B, $C(7, 7)$ and D. The side AB has equation $y = \frac{1}{2}x + 1$ and the length of $AB = 5\sqrt{5}$ units.

(a) Find the coordinates of B. [4]

(b) Prove that *ABCD* is a rectangle. [3]

(c) Calculate the area of *ABCD*. [2]

13 The equation of a curve is $y = \dfrac{6}{(2x-5)^3}$.

(a) Show that for $x > 2.5$, the curve has no stationary points. [3]

(b) The normal to the curve at $x = 1$ intersects another curve $36y = x^2 + 90x - 78$ at points A and B. Express the difference of the x-coordinates of A and B in the form $\sqrt{k}$, where k is an integer to be found. [7]

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