SWISS COTTAGE SECONDARY SCHOOL
SECONDARY FOUR AND FIVE
PRELIMINARY EXAMINATION

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Name: ____________________________________ (   )   Class: _________

ADDITIONAL MATHEMATICS
Paper 2

4049/02
Tuesday 10 September 2024
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

Questions	1	2	3	4	5	6
Marks						

This document consists of **18** printed pages and **2** blank pages.

Setter: Mrs Chen Yen Wah
Vetter: Mdm Zoe Pow

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Home of Thoughtful Leaders: Serve with Honour, Lead with Humility

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots\ldots + \binom{n}{r} a^{n-r} b^r + \ldots\ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

Section A (41 marks)

1 A man bought a new car. The value of the car depreciated with time so that its value, $\$P$, after t months' use is given by $P = 175\ 000e^{-kt}$, where k is a constant.

(a) Find the value of the car, $\$P$, when the man bought it. [1]

The value of the car is expected to be $\$162\ 000$ after eight months' use.

(b) Show that $k = 0.01$. [2]

(c) Use the result from **part (b)** to determine the age of the car correct to the nearest month, when its value reached half of the original value when the man bought it. [2]

2 A calculator must not be used in this question.

(a) Show that $\cot 15° = \sqrt{3} + 2$. [4]

(b) Use the result from **part (a)** to find an expression for $\operatorname{cosec}^2 15°$, in the form $p + q\sqrt{3}$ where p and q are integers. [2]

3 Given that $\int_0^m \left(2e^{2x} - \frac{5}{2}e^{-2x}\right) \mathrm{d}x = \frac{3}{4}$, where m is a positive constant,

(a) show that $4e^{4m} - 12e^{2m} + 5 = 0$. [3]

(b) Use the result from **part (a)** and a suitable substitution to find the value of m. [4]

4 The point A lies on the curve $y = x\ln x$. The tangent to the curve at A is parallel to the line $y = 2x + 3$.

(a) Find the exact coordinates of A. [4]

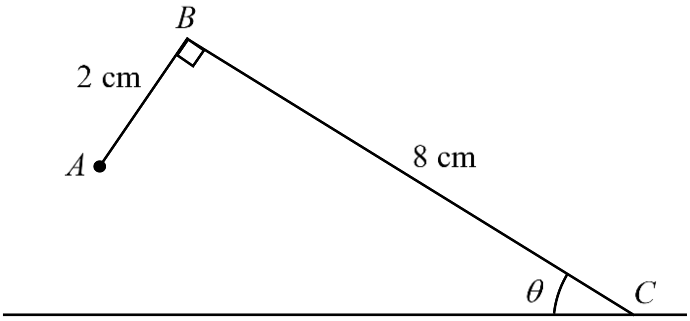
The normal to the curve $y = x\ln x$ at A meets the line $y = 2x + 3$ at the point B.

(b) Show that the x-coordinate of B is $k(e-2)$, where k is a constant to be found. [3]

5 **(a)** Prove the identity $\dfrac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x}=\tan x$. [3]

(b) Hence solve the equation $\dfrac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x}=3\cot 2x$ for $0° < x < 180°$. [4]

6



The diagram shows two rods AB and BC rigidly hinged at B so that angle $ABC = 90°$. The lengths of AB and BC are 2 cm and 8 cm respectively. The point C is fixed on horizontal ground and the rod BC rotates in a vertical plane with the rod BC inclined at an angle θ to the ground.

(a) Show that the height, h cm, of A above the ground is given by $h = a\sin\theta - b\cos\theta$, where a and b are integers to be found. [2]

(b) Using the values of a and b found in **part (a)**, express h in the form $R\sin(\theta-\alpha)$, where $R>0$ and $0\le\alpha\le\frac{\pi}{2}$. [4]

(c) Hence, state the maximum value of h and find the corresponding value of θ. [3]

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Questions	7	8	9	10	11
Marks					

Section B (49 marks)

7 The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 1 and the roots of $f(x)=0$ are $m,\ 2m$ and $(1-m)$, where $m>0$. It is given that $f(x)$ has a remainder of 30 when divided by $1-x$.

(a) Show that $2m^3-3m^2+m-30=0$. [3]

(b) Hence, find a value for m and show that there are no other real values of m which satisfy this equation. [4]

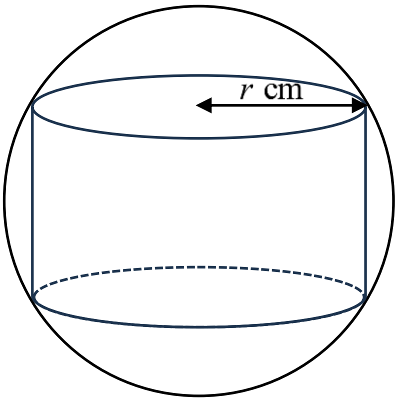
8 It is given that $y = \dfrac{x-1}{\sqrt{1+x}}$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ can be written in the form $\dfrac{x+3}{p\sqrt{(1+x)^3}}$, where p is a constant. [4]

(b) Given that y is changing at a consant rate of 0.6 units per second, find the rate of change of x when $x = 3$. [2]

(c) Use the result from **part (a)** to evaluate $\int_0^3 \frac{x+3}{3\sqrt{(1+x)^3}}\,dx$. [4]

9



The diagram shows a cylinder of radius r cm inscribed in a sphere with a fixed internal radius of 8 cm.

(a) Given that the curved surface area of the cylinder is S cm^2, show that

$$\frac{dS}{dr} = \frac{8\pi\left(32 - r^2\right)}{\sqrt{64 - r^2}}.$$

[5]

(b) Given that r varies, show that S has a stationary value when the height of the cylinder is equal to twice the radius of the cylinder. [3]

(c) Determine whether this value of S is a maximum or a minimum. [2]

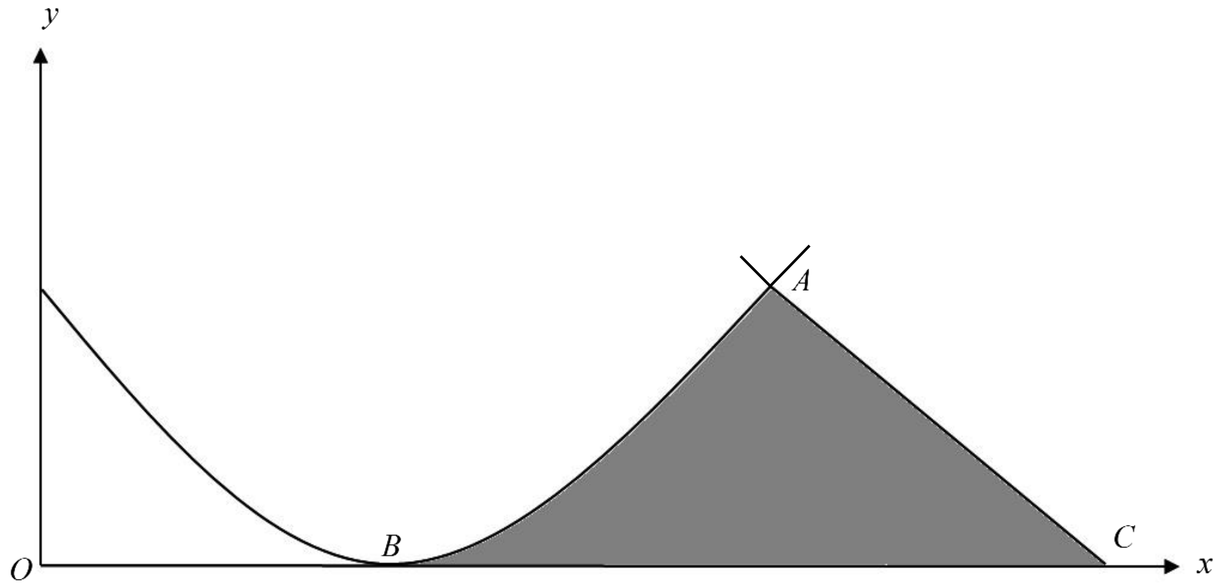
10 The equation of a circle is $x^2 + y^2 + 4x - 6y - 12 = 0$.

(a) Find the radius and coordinates of the centre of the circle. [4]

(b) Find the shortest distance of the centre of the circle to the line $y = 2x - 3$ and hence explain whether the circle intersects the line $y = 2x - 3$. [6]

Continuation of working space for Question **10(b).**

11



The diagram shows part of the curve $y = 1 - \sin x$ passing through the point A. The curve touches the x-axis at the point B.

The gradient of the tangent to the curve at A is 1 and the normal to the curve at A meet the x-axis at C.

Show that the area of the shaded region is $\frac{1}{2}(\pi - 1)$ units^2. [12]

Continuation of working space for question **11**.

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