Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

20 August 2024 Tuesday 2 hours 15 min

2024 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES
Write your name, index number and class in the spaces provided above.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided below the questions.
Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use											
Qn	1	2	3	4	5	6	7	8	9	10	*Marks Deducted*
Marks											

Category	Accuracy	Units	Symbols	Others
Question No.				

TOTAL MARKS
90

Setter: Mr Gregory Quek
Vetter: Mr Tan Lip Sing

This question paper consists of **21** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

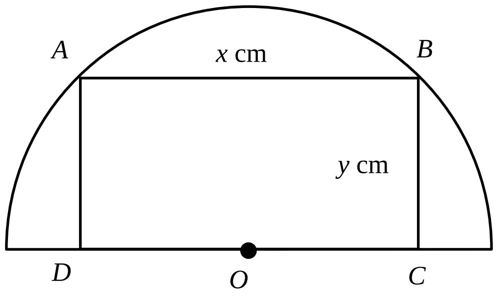
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Write down, and simplify, the first three terms in the expansion of $\left(3-\frac{2}{x}\right)^5$ in descending powers of x. [2]

(b) Given that there is no term independent of x in the expansion of $\left(5+ax^2\right)\left(3-\frac{2}{x}\right)^5$, hence find the value of the constant a. [3]

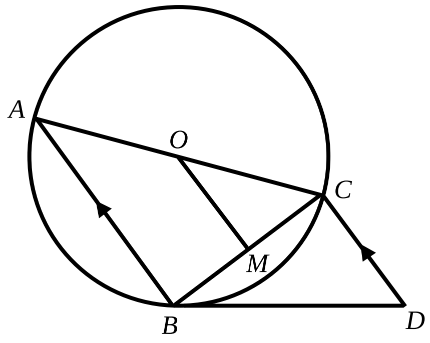
2 In the figure, $ABCD$ is a rectangle inscribed within a semicircle of radius 4 cm and centre O. It is given that $AB = x$ cm and $BC = y$ cm.



(a) Show that the area of the rectangle, A cm, is given by $A = \frac{1}{2}x\sqrt{64 - x^2}$. [2]

(b) Find the exact value of x for which A has a stationary value.
Give your answer in the form $k\sqrt{2}$, where k is an integer. [4]

3 The diagram shows a triangle *ABC* inscribed in the circle with centre *O*.
BD is a tangent to the circle at *B* and *AB* is parallel to *CD*. Point *M* is the midpoint of *BC*.



(a) Prove that triangles *ABC* and *BCD* are similar. [3]

(b) Prove that *ABMO* is a trapezium. [2]

(c) Prove that $OM = \dfrac{BC^2}{2CD}$. [3]

4 Milk is poured into an empty cup and heated. The temperature, T_m °C, of the milk in the cup, t minutes after it is heated, is modelled by the formula, $T_m = 5(2)^t + 20$.

(a) State the initial temperature of the milk. [1]

Coffee is poured into another empty cup. The temperature, T_c °C, of the coffee in the cup, t minutes after it is poured, is modelled by the formula, $T_c = 60(2)^{-t} + 25$.

(b) Find the time taken for the temperature of the coffee to drop to 35°C. [3]

(c) Find the time taken for the milk and the coffee to reach the same temperature. [4]

5 It is given that $f(x) = 2x^3 - x^2y - 13xy^2 - 6y^3$.

(a) Show that $x - 3y$ is a factor of $f(x)$. [2]

(b) If $y = 1$, find an expression in fully factorised form for $f(x)$. [3]

(c) Hence solve the equation $2e^{6z} - e^{4z} - 13e^{2z} - 6 = 0$ and show that the solution may be written in the form $\ln\sqrt{p}$, where p is an integer. [3]

6 **(a)** Given that $\tan\theta = 2\text{cosec}\,\theta$, show that $\cos^2\theta + 2\cos\theta - 1 = 0$. [3]

(b) Using **part (a)**, find the exact value of $\cos\theta$ in simplest form, given that $0° < \theta < 90°$. [3]

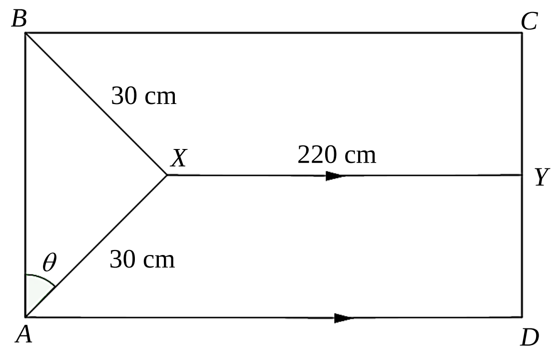
(c) Hence find the value of $\sec^2\theta$ in the form $a + b\sqrt{2}$, where a and b are integers. [5]

7 **(a)** Prove that $(\sin 2x)(\cot x) - 1 = \cos 2x$. [2]

(b) Given that $y = (\sin 2x)(\cot x) - 1$, hence show that $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 3\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) + 2y + 9\sin 2x = 0$ may be written in the form $\tan 2x = k$, where k is a constant to be found. [4]

(c) Solve $\tan 2x = -\sqrt{3}$ for $0 \le x \le 2\pi$, giving your answers in terms of π. [4]

8



The diagram shows a rectangular flag *ABCD*. *XAB* is a triangle with $AX = BX = 30$ cm and angle $XAB = \theta$ for $0 < \theta < 90°$. *XY* is parallel to *AD* and $XY = 220$ cm.

(a) Express the area of triangle *XAB* in the form $q \sin 2\theta$, where q is an integer. [2]

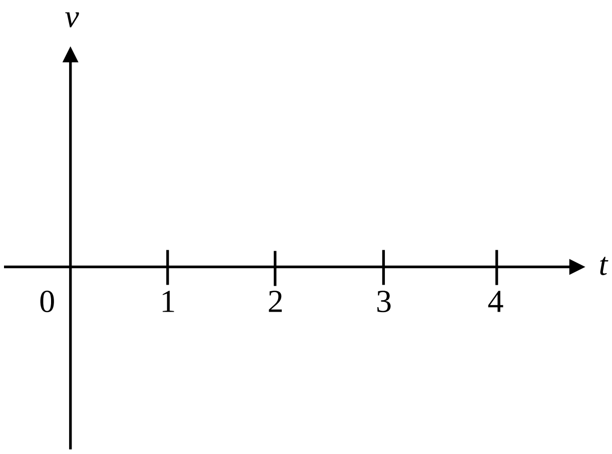
(b) Given that θ can vary, find the maximum possible area of triangle *XAB* and the value of θ at which this occurs. [2]

(c) Show that the perimeter, P cm, of the rectangular flag $ABCD$ can be expressed in the form $a\sin\theta + b\cos\theta + c$, where a, b and c are constants to be found. [3]

(d) By expressing P in the form $R\sin(\theta+\alpha)+c$, where $R > 0$ and $0 < \alpha < 90°$, explain if it is possible to have a flag with perimeter 550 cm. Show your working clearly. [5]

9 A particle moves in a straight line so that, t seconds after passing a fixed point O, its velocity, v metres per second, is given by $v = \pi \cos(\pi t) + \pi$.

(a) Sketch the velocity-time graph of the particle for $0 \le t \le 4$. [3]



(b) Determine how many times the particle is at instantaneous rest in the first 10 seconds. [1]

(c) Explain why the particle will never return to the origin O. [2]

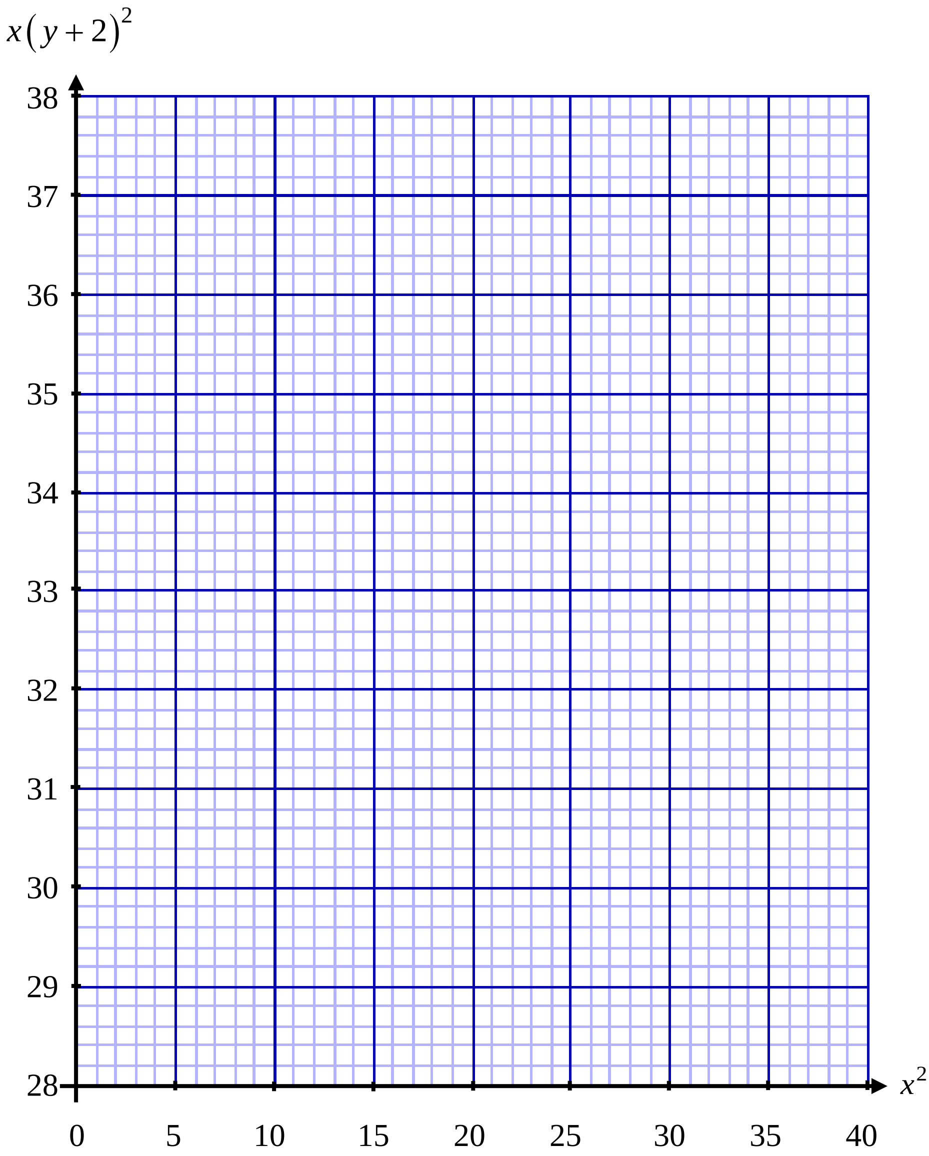
(d) Find an expression, in terms of *t*, for the displacement of the particle. [2]

(e) Calculate the average speed of the particle in the first 4 seconds. [3]

10 It is known that x and y are related by the equation $y = \sqrt{Ax + \frac{B}{x}} - 2$, where A and B are positive constants. The following table shows the values of the variables, x and y.

x	2	3	4	5	6
y	1.92	1.26	0.881	0.646	0.490

(a) Plot $x(y+2)^2$ against x^2 and draw a straight line graph to illustrate the information. [3]



(b) Express the equation $y = \sqrt{Ax + \frac{B}{x}} - 2$ in a form that will yield the straight line graph in **part (a)**. [2]

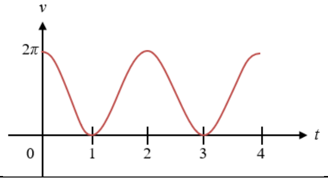
(c) Use your graph to estimate the value of A and of B. [2]

(d) Explain why the graph $y = \sqrt{Ax + \frac{B}{x}} - 2$ is undefined for $x \le 0$. [2]

(e) By drawing a suitable line on your graph, estimate the value of x for which $y + 2 = \frac{6}{\sqrt{x}}$.
Give your answer to 3 significant figures. [2]

END OF PAPER

Answer Key

1(a)	$\left(3-\frac{2}{x}\right)^5 = 243-\frac{810}{x}+\frac{1080}{x^2}+\dots$
1(b)	$a=-1.125$
2(b)	$x=4\sqrt{2}$
4(a)	$25°C$
4(b)	$t\approx 2.58$ min
4(c)	$t=2$ min
5(b)	$f(x)=(x-3)(2x+1)(x+2)$
5(c)	$z=\ln\sqrt{3}$
6(b)	$\cos\theta=-1+\sqrt{2}$
6(c)	$\sec^2\theta=3+2\sqrt{2}$
7(b)	$k=\frac{2}{3}$
7(c)	$x=\frac{\pi}{3},\frac{5\pi}{6},\frac{4\pi}{3},\frac{11\pi}{6}$
8(a)	Area of triangle $XAB=450\sin 2\theta$
8(b)	Maximum area of triangle $XAB=450\ \text{cm}^2$ Value of $\theta=45°$
8(c)	$P=60\sin\theta+120\cos\theta+440$
8(d)	$P=60\sqrt{5}\sin(\theta+63.434°)+440$ Yes, it is possible to have a flag with perimeter 550 cm when $\theta\approx 61.5°$. OR Since $500<P\le 574$, it is possible to have a flag with perimeter 550 cm.
9(a)	
9(b)	5 times
9(c)	Since $v\ge 0$, the **velocity** of the particle is **never negative**, hence the particle **does not change its direction of motion**. Therefore, the particle will never return to the origin *O*.
9(d)	$s=\sin(\pi t)+\pi t$
9(e)	Average speed = π m/s
10(b)	$x(y+2)^2=Ax^2+B$
10(c)	$A=gradient=0.2$ & $B=Y-\text{intercept}=30$
10(d)	When $x=0$, $\frac{B}{x}$ results in **division by zero error**. When $x<0$, since $A>0$ and $B>0$, $Ax+\frac{B}{x}<0$, hence $\sqrt{Ax+\frac{B}{x}}$ has **no real roots**. Hence, $y=\sqrt{Ax+\frac{B}{x}}-2$ is undefined for $x\le 0$.
10(e)	$x\approx 5.48$