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PRESBYTERIAN HIGH SCHOOL



ADDITIONAL MATHEMATICS Paper 2

4049/02

20 August 2024

Tuesday

2 hours 15 min

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2024 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

MARK SCHEME

**Q1 – 8 Mr Gregory Quek
Q9 – 10 Mr Tan Lip Sing**

- 1 (a) Write down, and simplify, the first three terms in the expansion of $\left(3 - \frac{2}{x}\right)^5$ in descending powers of x . [2]

$$\left(3 - \frac{2}{x}\right)^5 = 3^5 + 5(3)^4\left(-\frac{2}{x}\right) + \left(\frac{5}{2}\right)(3)^3\left(-\frac{2}{x}\right)^2 + \dots$$

$$\left(3 - \frac{2}{x}\right)^5 = 243 - \frac{810}{x} + \frac{1080}{x^2} + \dots$$

B2: Three correct terms

(B1: Two correct terms)

- (b) Given that there is no term independent of x in the expansion of $(5 + ax^2)\left(3 - \frac{2}{x}\right)^5$, hence find the value of the constant a . [3]

$$(5 + ax^2)\left(3 - \frac{2}{x}\right)^5 = (5 + ax^2)\left(243 - \frac{810}{x} + \frac{1080}{x^2} + \dots\right)$$

$$\text{Term independent of } x = (5)(243) + (ax^2)\left(\frac{1080}{x^2}\right)$$

M1: Derive terms indep. of x

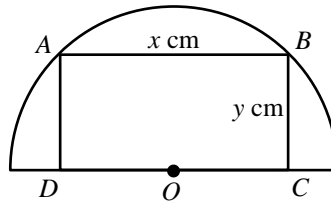
$$\Rightarrow 1215 + 1080a = 0$$

M1: Equate terms to zero

$$\Rightarrow a = -\frac{1215}{1080} = -1.125$$

A1: Accept $-1\frac{1}{8}$ or $-\frac{9}{8}$

- 2 In the figure, $ABCD$ is a rectangle inscribed within a semicircle of radius 4 cm and centre O . It is given that $AB = x$ cm and $BC = y$ cm.



- (a) Show that the area of the rectangle, A cm, is given by $A = \frac{1}{2}x\sqrt{64 - x^2}$. [2]

$$y^2 = 4^2 - \left(\frac{1}{2}x\right)^2$$

$$y = \sqrt{16 - \frac{1}{4}x^2}$$

M1: Correct application of Pythagoras Theorem

$$A = x\sqrt{16 - \frac{1}{4}x^2}$$

$$A = x\sqrt{\frac{1}{4}\sqrt{64 - x^2}}$$

M1: Factorise and simplify surd

$$A = \frac{1}{2}x\sqrt{64 - x^2}$$

(a.g.)

- (b) Find the exact value of x for which A has a stationary value.
Give your answer in the form $k\sqrt{2}$, where k is an integer. [4]

$$\frac{dA}{dx} = \frac{1}{2}x \cdot \left[\frac{1}{2}(64 - x^2)^{-\frac{1}{2}} \cdot (-2x) \right] + \sqrt{64 - x^2} \cdot \left(\frac{1}{2} \right)$$

M1, M1: Product rule

$$\frac{dA}{dx} = -\frac{1}{2}x^2(64 - x^2)^{-\frac{1}{2}} + \frac{1}{2}(64 - x^2)^{\frac{1}{2}}$$

$$\frac{dA}{dx} = \frac{1}{2}(64 - x^2)^{-\frac{1}{2}}[-x^2 + (64 - x^2)] = \frac{32 - x^2}{\sqrt{64 - x^2}}$$

For stationary value, $\frac{dA}{dx} = \frac{32 - x^2}{\sqrt{64 - x^2}} = 0$

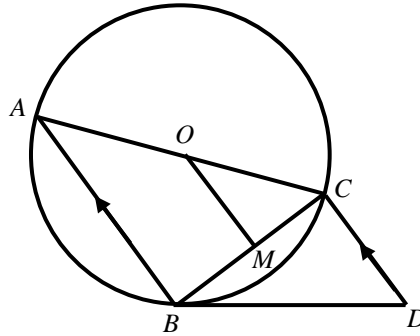
M1: Equate dA/dx to zero

$$32 - x^2 = 0$$

$$x = \sqrt{32} = 4\sqrt{2}$$

A1

- 3 The diagram shows a triangle ABC is inscribed in the circle with centre O . BD is a tangent to the circle at B and AB is parallel to CD . Point M is the midpoint of BC .



- (a) Prove that triangles ABC and BCD are similar. [3]

$$\angle ABC = \angle BCD \text{ (alt. } \angle s, AB \parallel CD) \quad \text{M1}$$

$$\angle BAC = \angle CBD \text{ (alternate segment theorem)}$$

$$\text{Triangles } ABC \text{ and } BCD \text{ are similar. (AA similarity)} \quad \text{A1}$$

- (b) Prove that $ABMO$ is a trapezium. [2]

Since O and M are the midpoints of AC and BC respectively,

$$OM \parallel AB \text{ (midpoint theorem)} \quad \text{M1}$$

$ABMO$ is a trapezium. (one pair of parallel sides) A1

- (c) Prove that $OM = \frac{BC^2}{2CD}$. [3]

$$\frac{AB}{BC} = \frac{BC}{CD} \text{ (corr. sides of similar } \Delta s) \quad \text{M1}$$

$$\text{Since } AB = 2OM \text{ (midpoint theorem)} \quad \text{M1}$$

$$\Rightarrow \frac{2OM}{BC} = \frac{BC}{CD}$$

$$\therefore OM = \frac{BC^2}{2CD} \quad \text{A1}$$

***Penalise 1m per question for any missing or incorrect reasons.**

- 4 Milk is poured into an empty cup and heated. The temperature, T_m °C, of the milk in the cup, t minutes after it is heated, is modelled by the formula, $T_m = 5(2)^t + 20$.

(a) State the initial temperature of the milk. [1]

Initial temperature of milk = $5(2)^0 + 20 = 25^\circ\text{C}$	B1
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Coffee is poured into another empty cup. The temperature, T_c °C, of the coffee in the cup, t minutes after it is poured, is modelled by the formula, $T_c = 60(2)^{-t} + 25$.

(b) Find the time taken for the temperature of the coffee to drop to 35°C . [3]

$60(2)^{-t} + 25 = 35$ $(2)^{-t} = \frac{35 - 25}{60} = \frac{1}{6}$ $\lg(2)^{-t} = \lg\left(\frac{1}{6}\right)$ $-t \lg(2) = \lg\left(\frac{1}{6}\right)$ $t = -\lg\left(\frac{1}{6}\right) \div \lg(2) = 2.5849$ $t \approx 2.58 \text{ min (3sf)}$	M1: Isolate $(2)^{-t}$ M1: Take lg on both sides A1
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(c) Find the time taken for the milk and the coffee to reach the same temperature. [4]

$5(2)^t + 20 = 60(2)^{-t} + 25$ $5(2)^{2t} + 20(2)^t = 60 + 25(2)^t$ $5(2)^{2t} - 5(2)^t - 60 = 0$ $(2)^{2t} - (2)^t - 12 = 0$ Let $u = (2)^t$, $u^2 - u - 12 = 0$ $(u - 4)(u + 3) = 0$ $u = 4 \text{ or } u = -3(\text{rejected})$ $(2)^t = 4$ $t = 2 \text{ min}$	M1: Equate T_m to T_c M1: Multiply 2^t throughout / obtain quad. eqn. <div style="border: 1px solid black; padding: 10px; margin: 10px auto; width: fit-content;"> Let $u = (2)^t$, $5u + 20 = \frac{60}{u} + 25$ $5u^2 - 5u - 60 = 0$ $u^2 - u - 12 = 0$ </div> M1: Solve quadratic equation A1
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5 It is given that $f(x) = 2x^3 - x^2y - 13xy^2 - 6y^3$.

(a) Show that $x - 3y$ is a factor of $f(x)$. [2]

$$f(3y) = 2(3y)^3 - (3y)^2y - 13(3y)y^2 - 6y^3$$

$$f(3y) = 54y^3 - 9y^3 - 39y^3 - 6y^3 = 0$$

M1: Sub. into $f(x)$ & simplify

Since $f(3y) = 0$, by Factor Theorem, $x - 3y$ is a factor of $f(x)$. AG1

(b) If $y = 1$, find an expression in fully factorised form for $f(x)$. [3]

$$\text{Let } f(x) = 2x^3 - x^2 - 13x - 6$$

$$= (x - 3)[2x^2 + bx + 2]$$

Comparing x^2 term:

$$-1 = b + (-3)(2)$$

$$b = 5$$

M1: Comparing coefficient
(or long division)

$$\Rightarrow f(x) = (x - 3)[2x^2 + 5x + 2]$$

A1

$$\Rightarrow f(x) = (x - 3)(2x + 1)(x + 2)$$

A1

(c) Hence solve the equation $2e^{6z} - e^{4z} - 13e^{2z} - 6 = 0$ and show that the solution may be written in the form $\ln \sqrt{p}$, where p is an integer. [3]

$$\text{Let } x = e^{2z},$$

$$\text{we get } 2e^{6z} - e^{4z} - 13e^{2z} - 6 = 0$$

$$\Rightarrow (e^{2z} - 3)(2e^{2z} + 1)(e^{2z} + 2) = 0$$

M1: Sub. $x = e^{2z}$ into (b)

$$\Rightarrow e^{2z} = 3 \quad \text{or} \quad 2e^{2z} = -1 (\text{rejected}) \quad \text{or} \quad e^{2z} = -2 (\text{rejected}) \quad \text{A1: Seen } e^{2z} = 3$$

$$\ln e^{2z} = \ln 3$$

$$2z = \ln 3$$

$$z = \frac{1}{2} \ln 3$$

$$\therefore z = \ln \sqrt{3}$$

A1

- 6 (a) Given that $\tan \theta = 2\operatorname{cosec} \theta$, show that $\cos^2 \theta + 2\cos \theta - 1 = 0$. [3]

$$\begin{aligned}\tan \theta &= 2\operatorname{cosec} \theta \\ \frac{\sin \theta}{\cos \theta} &= \frac{2}{\sin \theta} && \text{M1: Seen either one} \\ \sin^2 \theta &= 2\cos \theta \\ 1 - \cos^2 \theta &= 2\cos \theta && \text{M1: Apply Pythagorean identity} \\ \cos^2 \theta + 2\cos \theta - 1 &= 0 && \text{AG1}\end{aligned}$$

- (b) Using **part (a)**, find the exact value of $\cos \theta$ in simplest form, given that $0^\circ < \theta < 90^\circ$. [3]

$$\begin{aligned}\cos^2 \theta + 2\cos \theta - 1 &= 0 \\ \cos \theta &= \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)} && \text{M1: Apply quadratic formula} \\ \cos \theta &= \frac{-2 \pm \sqrt{8}}{2} \\ \cos \theta &= -1 \pm \sqrt{2} && \text{M1: Attempt to simplify} \\ \text{Since } 0^\circ < \theta < 90^\circ, \cos \theta &\text{ must be positive.} \\ \therefore \cos \theta &= -1 + \sqrt{2} && \text{A1}\end{aligned}$$

- (c) Hence find the value of $\sec^2 \theta$ in the form $a + b\sqrt{2}$, where a and b are integers. [5]

$$\begin{aligned}\sec^2 \theta &= \frac{1}{\cos^2 \theta} && \text{M1: Seen } \frac{1}{\cos^2 \theta} \\ \sec^2 \theta &= \frac{1}{(-1 + \sqrt{2})^2} \\ \sec^2 \theta &= \frac{1}{(\sqrt{2})^2 - 2(\sqrt{2})(1) + 1^2} && \text{M1: Expand the denominator} \\ \sec^2 \theta &= \frac{1}{3 - 2\sqrt{2}} \times \frac{3 + 2\sqrt{2}}{3 + 2\sqrt{2}} && \text{M1: Rationalise the denominator} \\ \sec^2 \theta &= \frac{3 + 2\sqrt{2}}{3^2 - (2\sqrt{2})^2} && \text{M1: Simplify the denominator} \\ \sec^2 \theta &= 3 + 2\sqrt{2} && \text{A1}\end{aligned}$$

- 7 (a) Prove that $(\sin 2x)(\cot x) - 1 = \cos 2x$. [2]

$$\begin{aligned}
 LHS &= (\sin 2x)(\cot x) - 1 \\
 &= (2 \sin x \cos x) \left(\frac{\cos x}{\sin x} \right) - 1 && \text{M1: Seen } 2\sin x \cos x \\
 &= 2 \cos^2 x - 1 && \text{M1} \\
 &= \cos 2x = RHS && \text{(a.g)}
 \end{aligned}$$

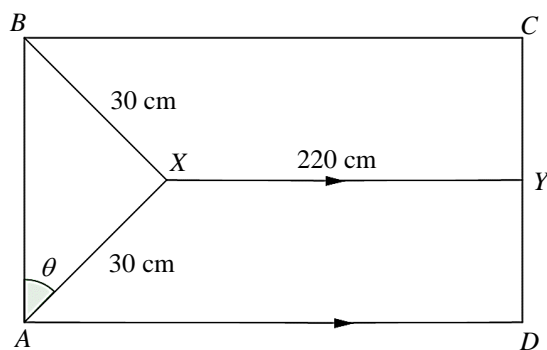
- (b) Given that $y = (\sin 2x)(\cot x) - 1$, hence show that $\frac{d^2 y}{dx^2} + 3\left(\frac{dy}{dx}\right) + 2y + 9 \sin 2x = 0$ may be written in the form $\tan 2x = k$, where k is a constant to be found. [4]

$$\begin{aligned}
 y &= (\sin 2x)(\cot x) - 1 = \cos 2x \\
 \frac{dy}{dx} &= -2 \sin 2x && \text{B1} \\
 \frac{d^2 y}{dx^2} &= -4 \cos 2x && \text{B1} \\
 \frac{d^2 y}{dx^2} + 3\left(\frac{dy}{dx}\right) + 2y + 9 \sin 2x &= 0 \\
 -4 \cos 2x + 3(-2 \sin 2x) + 2 \cos 2x + 9 \sin 2x &= 0 && \text{M1: Correct substitution} \\
 3 \sin 2x &= 2 \cos 2x \\
 \tan 2x &= \frac{2}{3} \\
 \therefore k &= \frac{2}{3} && \text{A1}
 \end{aligned}$$

- (c) Solve $\tan 2x = -\sqrt{3}$ for $0 \leq x \leq 2\pi$, giving your answers in terms of π . [4]

$$\begin{aligned}
 \alpha &= \tan^{-1}(\sqrt{3}) = \frac{\pi}{3} && \text{M1: Find reference angle} \\
 2x &= \pi - \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, 3\pi - \frac{\pi}{3}, 4\pi - \frac{\pi}{3} && \text{M1: Find angles in 2nd and 4th quadrants} \\
 2x &= \frac{2\pi}{3}, \frac{5\pi}{3}, \frac{8\pi}{3}, \frac{11\pi}{3} \\
 x &= \frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6} && \text{A2: One mark for each correct pair of angles}
 \end{aligned}$$

*Penalise 1m for answers not in terms of π .



The diagram shows a rectangular flag $ABCD$. XAB is a triangle with $AX = BX = 30$ cm and angle $XAB = \theta$ for $0 < \theta < 90^\circ$. XY is parallel to AD and $XY = 220$ cm.

- (a) Express the area of triangle XAB in the form $q \sin 2\theta$, where q is an integer. [2]

Area of triangle $XAB = \frac{1}{2}(30)(30)\sin(180^\circ - 2\theta)$	M1: Apply formula $\frac{1}{2}bc\sin A$
Area of triangle $XAB = 450 \sin 2\theta$	A1

- (b) Given that θ can vary, find the maximum possible area of triangle XAB and the value of θ at which this occurs. [2]

This occurs when $\sin 2\theta = 1$,

Maximum area of triangle $XAB = 450 \text{ cm}^2$ B1: F.T.

Value of $\theta = 45^\circ$ B1

- (c) Show that the perimeter, P cm, of the rectangular flag $ABCD$ can be expressed in the form $a \sin \theta + b \cos \theta + c$, where a , b and c are constants to be found. [3]

$AD = 30 \sin \theta + 220$ $AB = 2 \times 30 \cos \theta$	M1: Either AD or AB
Perimeter = $2[30 \sin \theta + 220] + 2[2 \times 30 \cos \theta]$	M1: Attempt to find perimeter
$P = 60 \sin \theta + 120 \cos \theta + 440$	A1

- (d) By expressing P in the form $R \sin(\theta + \alpha) + c$, where $R > 0$ and $0 < \alpha < 90^\circ$, explain

if it is possible to have a flag with perimeter 550 cm. Show your working clearly. [5]

$$R = \sqrt{60^2 + 120^2} = \sqrt{18000} = 60\sqrt{5}$$

$$\text{M1: Seen } R = \sqrt{a^2 + b^2}$$

$$\alpha = \tan^{-1}\left(\frac{120}{60}\right) = 63.434^\circ$$

$$\text{M1: Seen } \alpha = \tan^{-1} \frac{b}{a}$$

$$P = 60\sqrt{5} \sin(\theta + 63.434^\circ) + 440$$

Method 1

$$\text{Let } 60\sqrt{5} \sin(\theta + 63.434^\circ) + 440 = 550$$

$$\sin(\theta + 63.434^\circ) = \frac{550 - 440}{60\sqrt{5}}$$

$$\text{Reference angle} = \sin^{-1}\left(\frac{11}{6\sqrt{5}}\right) = 55.0739$$

M1: Find reference angle

$$\theta + 63.434^\circ = 55.0739^\circ \text{ or } 180^\circ - 55.0739^\circ$$

M1: Find θ in 1st & 2nd quad

$$\theta = -8.3601^\circ (\text{rejected}) \text{ or } 61.4921^\circ$$

Yes, it is possible to have a flag with perimeter 550 cm when $\theta \approx 61.5^\circ$ (1dp) A1

Method 2

$$\text{Maximum } P = 60\sqrt{5} + 440 = 574 \text{ cm}$$

M1

$$\text{When } \theta = 90^\circ, \text{ Minimum } P = 60\sqrt{5} \sin(90^\circ + 63.434^\circ) + 440 = 500 \text{ cm}$$

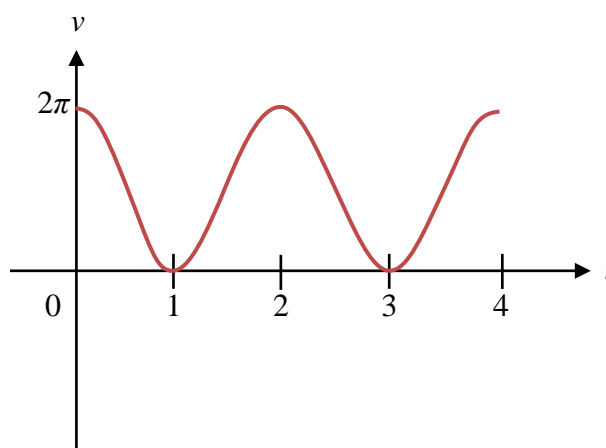
M1

Since $500 < P \leq 574$, it is possible to have a flag with perimeter 550 cm. A1

- 9 A particle moves in a straight line so that, t seconds after passing a fixed point O , its velocity, v metres per second, is given by $v = \pi \cos(\pi t) + \pi$.

- (a) Sketch the velocity-time graph of the particle for $0 \leq t \leq 4$.

[3]



B1: Correct shape

B1: Correct max & min values

B1: Correct period

- (b) Determine how many times the particle is at instantaneous rest in the first 10 seconds. [1]

From the graph, the particle is at instantaneous rest when $v = 0$ at every odd second.

Therefore, there are **5 times** in the first 10 seconds. B1

- (c) Explain why the particle will never return to the origin O .

[2]

Since $v \geq 0$, the **velocity** of the particle is **never negative**, B1
 hence the particle **does not change its direction of motion**. B1
 Therefore, the particle will never return to the origin O . (a.g)

- (d) Find an expression, in terms of t , for the displacement of the particle.

[2]

$$s = \int \pi \cos(\pi t) + \pi \, dt$$

$$s = \frac{\pi \sin(\pi t)}{\pi} + \pi t + c$$

M1: Apply integration

When $t = 0$, $s = 0$, thus $c = 0$.

(e) Calculate the average speed of the particle in the first 4 seconds.

[3]

When $t = 0$, $s = 0$.

When $t = 4$, $s = \sin(4\pi) + 4\pi = 4\pi$

M1: Find displacement at $t = 4$

Average speed = $\frac{4\pi}{4}$

M1: Find average speed

Average speed = π m/s

A1

- 10 It is known that x and y are related by the equation $y = \sqrt{Ax + \frac{B}{x}} - 2$, where A and B are positive constants. The following table shows the values of the variables, x and y .

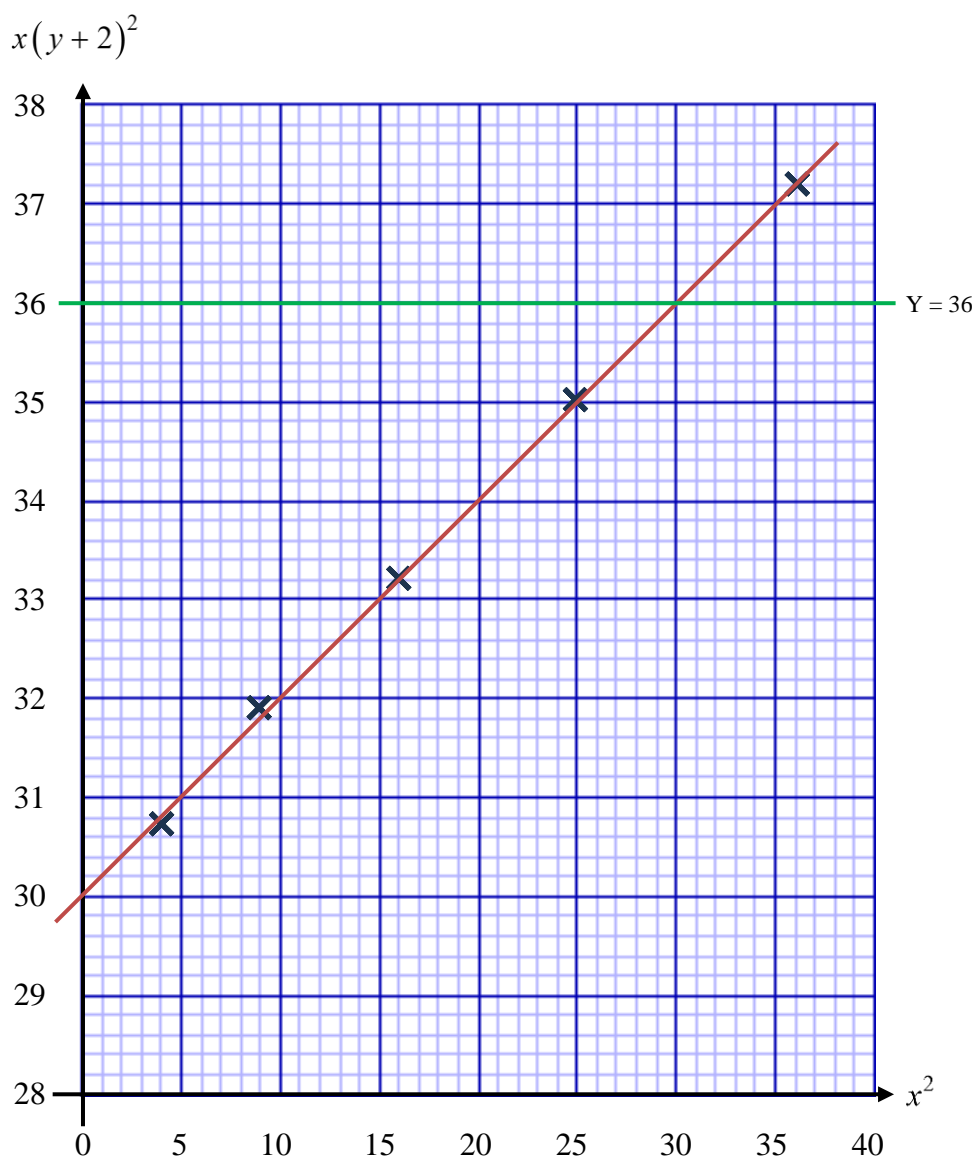
x	2	3	4	5	6
y	1.92	1.26	0.881	0.646	0.490

x^2	4	9	16	25	36
$x(y+2)^2$	30.7	31.9	33.2	35.0	37.2

- (a) Plot $x(y+2)^2$ against x^2 and draw a straight line graph to illustrate the information. [3]

B2: All correct points plotted (B1: at least 3 correct)

B1: Best fit line



- (b) Express the equation $y = \sqrt{Ax + \frac{B}{x}} - 2$ in a form that will yield the straight line graph in part (a). [2]

$$y = \sqrt{Ax + \frac{B}{x}} - 2$$

$$y + 2 = \sqrt{Ax + \frac{B}{x}}$$

$$(y + 2)^2 = Ax + \frac{B}{x} \quad \text{M1: Taking square on both sides}$$

$$x(y + 2)^2 = Ax^2 + B \quad \text{A1}$$

- (c) Use your graph to estimate the value of A and of B . [2]

$$A = \text{gradient} = \frac{37.2 - 33.2}{36 - 16} = 0.2 \quad \text{B1: Accept +/- 0.01}$$

$$B = Y - \text{intercept} = 30 \quad \text{B1: Accept +/- 0.5}$$

- (d) Explain why the graph $y = \sqrt{Ax + \frac{B}{x}} - 2$ is undefined for $x \leq 0$. [2]

When $x = 0$, $\frac{B}{x}$ results in **division by zero error**. B1

When $x < 0$, since $A > 0$ and $B > 0$, $Ax + \frac{B}{x} < 0$, hence $\sqrt{Ax + \frac{B}{x}}$ has **no real roots**. B1

Hence, $y = \sqrt{Ax + \frac{B}{x}} - 2$ is undefined for $x \leq 0$. (a.g.)

- (e) By drawing a suitable line on your graph, estimate the value of x for which $y + 2 = \frac{6}{\sqrt{x}}$.
Give your answer to 3 significant figures. [2]

$$y + 2 = \frac{6}{\sqrt{x}}$$

$$(y + 2)^2 = \frac{36}{x} \quad \text{B1: Draw } Y = 36 \text{ on the same axes}$$

$$x(y + 2)^2 = 36$$

From the graph, when $x(y + 2)^2 = 36$, $x^2 = 30$

$$\therefore x = \sqrt{30} = 5.4772 \approx 5.48 \text{ (3sf)} \quad \text{B1: Accept +/- 0.1}$$