1   Find the range of values of the constant m for which the curve $y = (m - 6)x^2 - 8x + m$ lies completely above the x-axis.   [4]

2   Show that the line $x + y = m$ will intersect the curve $x^2 + 2y^2 = 2x + 3$ if $m^2 \leq 2m + 5$.   [4]

3   It is given that $f(x) = (b - 3x)e^{2-3x}$. Find the value of the constant b if f(x) is a decreasing function when $x < \frac{4}{3}$.   [4]

4   Prove that $\dfrac{2-\operatorname{cosec}^2\theta}{\operatorname{cosec}^2\theta + 2\cot\theta} = \dfrac{1-\cot\theta}{1+\cot\theta}$.   [4]

5   Express $\dfrac{4x^2+5x-32}{(x+2)(x^2-9x-22)}$ in partial fractions.   [5]

6 **(a)** Show that $3 \cos x = 2 \operatorname{cosec} x \cot x$ can be written as $\cos x (3\sin^2 x - 2) = 0$. [3]

(b) Hence, solve the equation $3 \cos(0.6y - 1.4) = 2 \operatorname{cosec}(0.6y - 1.4) \cot(0.6y - 1.4)$

for values of y between -3 and 4. [5]

7 (a) Given that $y = \dfrac{1+\sin x}{\cos x}$, find $\dfrac{dy}{dx}$. [2]

(b) Hence, without using a calculator, find the value of each of the constants p and q for which $\displaystyle\int_0^{\frac{\pi}{3}} \dfrac{3 + 3\sin x - 10\cos^3 x}{5\cos^2 x}\, dx = p + q\sqrt{3}$. [6]

8   The height of Jeremiah above the surface of the water, h metres, can be modelled by the equation $h = -4.9t^2 + 8t + 5$, where t is the time in seconds after he leaves the diving board.

(a)   State the height of the diving board above the surface of the water.   [1]

(b)   Express h in the form $k - a(x - b)^2$, where k, a and b are constants to be determined.   [3]

(c)   State the greatest height reached by Jeremiah and the corresponding time when the greatest height occurs.   [2]

(d)   Using your answer obtained in (b), calculate the duration which Jeremiah stay in the air.   [2]

9   Solve the simultaneous equations.

$$\frac{5^P}{25} = 125^q$$

$$\log_3 7 = 1 + \log_3(11q - 2p)$$

10   The equation of a curve is $y = x^3 \ln x$.

(a)   Find the **exact** coordinates of the stationary point(s) of the curve.   [4]

(b)   Determine the nature of the stationary point(s) of the curve.   [2]

A particle moves along the curve $y = x^3 \ln x$. At point M, the x-coordinate of the particle is increasing at a rate of 0.06 units/s and the y-coordinate is increasing at a rate of $0.3x^2$ units/s.

**(c)** Find the exact coordinate of M. [4]

11   A circle, C_1, has equation $x^2 + y^2 + kx - 6y = h$, where k and h are constants.

(a)   Given that the radius C_1 of is 7 units and the coordinates of the centre is $(-2, m)$, find the values of k, m and h.   [4]

(b)   Another circle, C_2, has diameter PQ. The point P is $(3, n)$ and Q is $(-5, 5)$. The equation PQ is $4y + 3x = 5$.

(i)   Find the equation of C_2.   [4]

(ii) Explain, with appropriate working, why the point $S\ (4, 5)$ only lies inside the circle C_1 but not C_2. [2]

12 The diagram shows a parallelogram $ABCD$ in which D lies on the x-axis. Point A is $(-1, 4)$, B $(2, 7)$ and C $(4, 3)$. The point E lies on BC such that $5BE = 2BC$.
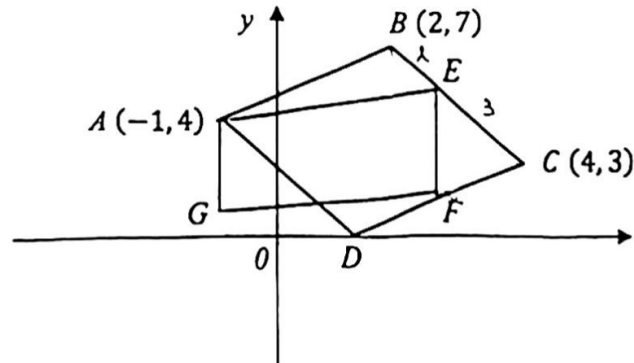
(a) Find the coordinates of D, E and F.   [8]

Continuation of working space for question **12(a)**.

(b) AG and EF are two vertical lines. The y-coordinate of G is $\frac{2}{5}$. E and F lie on BC and DC respectively. Explain, with an appropriate working, what is the name of the special quadrilateral $AEFG$.   [2]

13   A container in the shape of a right pyramid, has a height of 45 cm and a square base of side 20 cm, was initially empty. Sand is then allowed to flow into the container through a small hole at the top. After t seconds, the height of the sand in the container is $(45 - x)$ cm and the volume of the sand in the container is V cm^3.

(a)   Show that $V = 6000 - \dfrac{16}{243}x^3$.   [3]

$45 - x$

$t-1$

(b)   Given that the rate of flow of the sand into the container is bx^2 cm³/s, where b is a constant. Find the **numerical** value of the rate of change of x if the height of the sand in the container is 36 cm after 24 seconds.   [7]

End of paper