1 **(a)** Given that there is a term that is independent of x in the expansion of $\left(5x^2 - \frac{1}{\sqrt{x}}\right)^n$, where n is a positive integer, find the smallest possible value of n. [3]

(b) Using the value of n found in part **(a)**, explain if there is any term independent of x in the expansion of $\left(1-\frac{1}{50x^5}\right)\left(5x^2-\frac{1}{\sqrt{x}}\right)^n$. [4]

2 The expression $10\,\mathrm{f}(x) + 3\,\mathrm{f}'(x) - \mathrm{f}''(x) + 7\sin 2x + 3\cos 2x$, may be written as $10x + 43$, when $\mathrm{f}'(x) = e^{5x} + 2\sin^2 x$. Find $\mathrm{f}(x)$.

[6]

3 **Do not use a calculator in this question.**

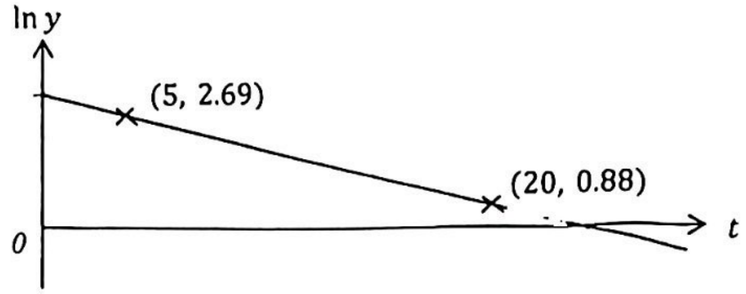
It is given that $\tan A = \frac{5}{12}$ and that $\frac{\pi}{2} < A < \frac{3\pi}{2}$.

(a) By expressing $\cos 3A = \cos(2A + A)$, find the exact value of $\cos 3A$. [4]

(b) Find the exact value of $\tan\frac{A}{2}$. [4]

4 Coffee is poured into an empty cup. At time t minutes after the coffee is poured, its temperature exceeds room temperature by y°C. The room temperature is 25°C.

(a)



The variables t and y are related by the equation $y = e^{kt+c}$, where k and c are constants. The diagram above shows the straight line graph obtained by plotting $\ln y$ against t. The line passes through the points (5, 2.69) and (20, 0.88).
Find the value of k and of c. [3]

(b) Calculate the time which the temperature of the coffee would drop to half of its initial temperature. [3]

(c) At the same time, when the coffee was poured into the cup, coffee of the same volume is also poured into an empty tumbler.

Similarly, at time t minutes after the coffee is poured into the tumbler, its temperature exceeds room temperature by y°C and is modelled by another equation.

The solution to the equation $e^{(k+0.2)t} = e^{5-c}$ is the timing where the temperature of the coffee in both the cup and tumbler are equivalent. By using the diagram in part **(a)**, outline the steps to find this timing. [4]

5

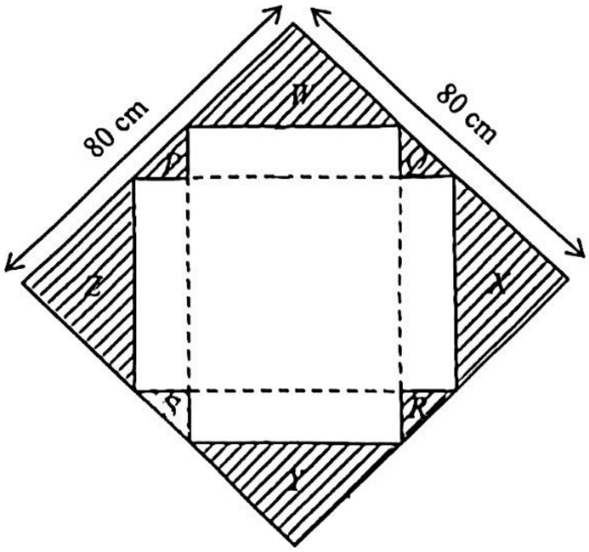


Diagram 1

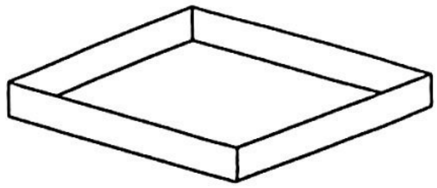


Diagram 2

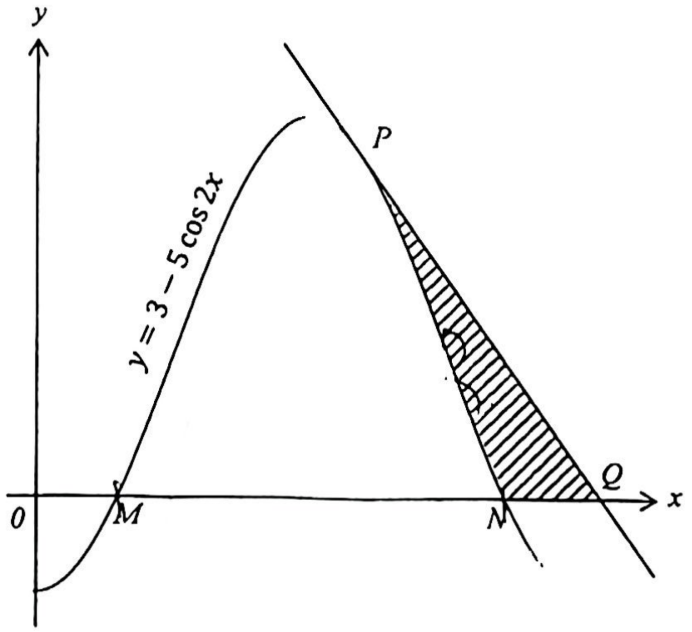
Diagram 1 shows a piece of square cardboard of side 80 cm.
Four small identical isosceles triangles, P, Q, R and S and four big identical isosceles triangles, W, X, Y and Z are removed. The remaining cardboard is folded along the dotted lines to form an open container as shown in Diagram 2.

(a) Let the height of the open container be h cm, show that the total exterior area, A cm^2, of the open container is $\cancel{3200} - 80\sqrt{2}h + h^2$. [4]

(b) Find the value of h for which the total exterior area of the open container is a maximum. [3]

(c) Hence, find the maximum total exterior area of the open container that can be obtained from the piece of square cardboard. [2]

6



The diagram shows part of the curve $y = 3 - 5\cos 2x$, which cuts the x-axis at M and N. The tangent to the curve at P is -5 and this tangent cuts the x-axis at Q. Find the area of the shaded region PNQ. [9]

Continuation of working space for Question 6.

7 (a) Show that $x + 2y$ is a factor of $4x^3 + x^2y - 11xy^2 + 6y^3$ and hence factorise $4x^3 + x^2y - 11xy^2 + 6y^3$ completely. [3]

(b) Hence, solve the equation $4^{p+1} + 2^{p+1} = 44 - 48(2^{-p})$. [5]

8 **(a)** The graph of $y = a \sin bx + c$ has one minimum point at $\left(\frac{\pi}{6}, 1\right)$ and the next maximum point after this has coordinates $\left(\frac{\pi}{2}, 9\right)$. Find the values of the constants a, b and c. [3]

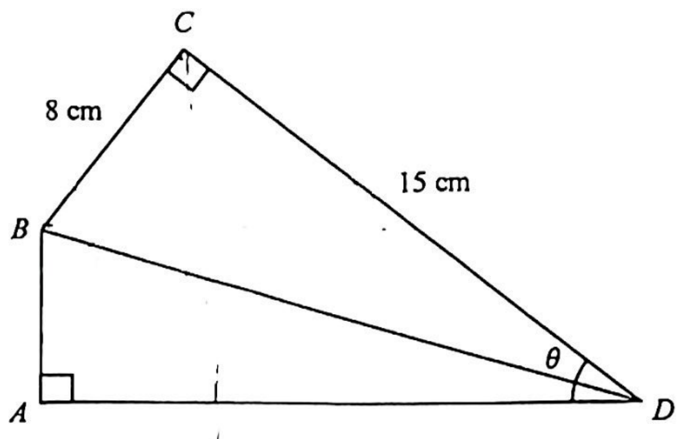
(b) A particle, travelling in a straight line, passes through a fixed point O. The velocity, v m/s, at time t seconds, is given by $v = 3t^2 - 5t + 7$ for $0 \leq t \leq 5$.

(i) Find the acceleration of the particle when $t = 4$. [2]

After $t > 5$ seconds, the particle travels at a velocity, in m/s, where $v = -4t + 77$.

(ii) Find the total distance travelled by the particle in the first 30 seconds. [7]

9



The diagram shows a metal structure $ABCD$ consisting of five metal rods of different lengths. T~~he length of BC and CD are~~ 15 m and ~~8 m respectively~~. Angle $ADC = \theta$ for $0° < \theta < 90°$.

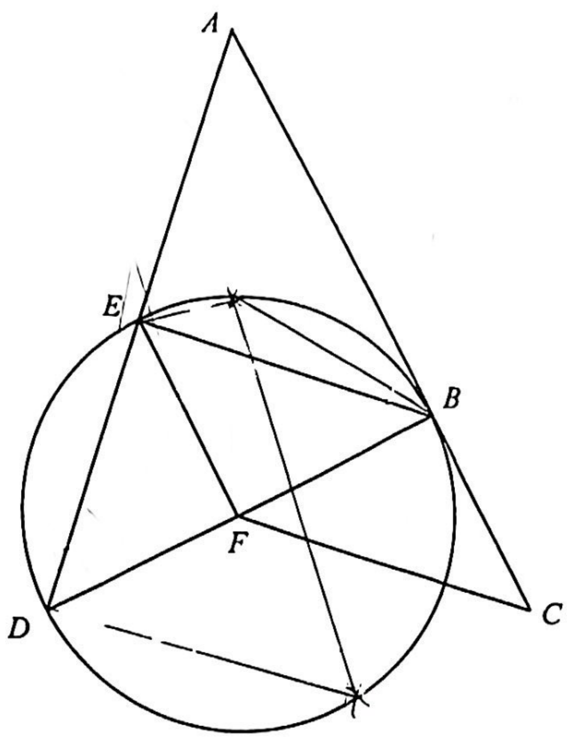
(a) Show that the total lengths, P m, of the five metal rods used is $40 + 23\sin\theta + 7\cos\theta$. [3]

(b) Express P in the form $40 + R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [2]

(c) Find the value of θ if the total length of the five metal rods is 60 m. [3]

(d) State the minimum value of $\frac{1}{40+(R\sin(\theta+\alpha))^2}$ and the corresponding value of θ for which it occurs. [3]

10



The diagram shows a circle, centre F, with BD as diameter. The point E lies on the circle. The tangent at a point B on the circle meets DE extended at the point A. Point C lies on AB extended and $ED = AE$.

(a) State the relationship between the length of DF and AB. Give reasons to support your answer. [2]

(b) Prove that

(i) triangle ABE is similar to triangle EDF, [2]

(ii) $AD^2 - BD^2 = 2 \times BE \times ED$. [3]

Point G is on minor arc EB such that BG bisects angle EBA.
Point H is on the circle such that $EGHD$ is a cyclic quadrilateral.

(c) Prove that angle $GHD = 90° -$ angle GBE. [3]

-End of Paper-