Qn	Solutions	
1	$m-6>0$ and $m>6$ Hence, $m>8$	$b^2-4ac<0$ $(-8)^2-4(m-6)(m)<0$ $64-4m^2+24m<0$ $m^2-6m-16>0$ $(m+2)(m-8)>0$ $m<-2$ or $m>8$
2	$y=m-x$ ....(1) Sub (1) into $x^2+2y^2=2x+3$, $x^2+2(m-x)^2-2x-3=0$ $3x^2-x(4m+2)+2m^2-3=0$ Discriminant $=[-(4m+2)]^2-4(3)(2m^2-3)$ $=16m^2+16m+4-24m^2+36$ $=-8(m^2-2m-5)$ Given $m^2\le 2m+5 \Rightarrow m^2-2m-5\le 0$ Hence $-8(m^2-2m-5)\ge 0$ Since, discriminant ≥ 0, hence the line will intersect the curve.	
3	$f(x)=(b-3x)\,e^{2-3x}$ $f'(x)=-3e^{2-3x}+(b-3x)(-3e^{2-3x})$ $=-3e^{2-3x}(1+b-3x)$ $=3e^{2-3x}(3x-1-b)$ For decreasing function, $3e^{2-3x}(3x-1-b)<0$ Since $3e^{2-3x}>0 \Rightarrow 3x-1-b<0$ $x<\frac{1+b}{3}$ Given $x<\frac{4}{3}$, $\therefore 1+b=4$ $b=3$	

4	LHS $=\frac{2-\operatorname{cosec}^2\theta}{\operatorname{cosec}^2\theta+2\cot\theta}$ $=\frac{2-(1+\cot^2\theta)}{1+\cot^2\theta+2\cot\theta}$ $=\frac{1-\cot^2\theta}{(1+\cot\theta)^2}$ $=\frac{(1+\cot\theta)(1-\cot\theta)}{(1+\cot\theta)^2}$ $=\frac{1-\cot\theta}{1+\cot\theta}$	
5	$x^2-9x-22=(x-11)(x+2)$ $\frac{4x^2+5x-32}{(x-11)(x+2)^2}=\frac{A}{x-11}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$ $4x^2+5x-3=A(x+2)^2+B(x-11)(x+2)+C(x-11)$ Let $x=11$, $4(11)^2+5(11)-32=A(13)^2$ $A=3$ Let $x=-2$, $4(-2)^2+5(-2)-32=C(-13)$ $C=2$ Let $x=0$, $-32=A(4)+B(-11)(2)+C(-11)$ $-32=12-22B-22$ $B=1$ $\frac{3}{x-11}+\frac{1}{x+2}+\frac{2}{(x+2)^2}$	
6(a)	$3\cos x=2\left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right)$ $3\cos x\sin^2 x=2\cos x$ $3\cos x\sin^2 x-2\cos x=0$ $\cos x(3\sin^2 x-2)=0$ (Shown)	

(b) From (a), $\cos x(3\sin^2 x - 2) = 0$

$3\sin^2 x - 2 = 0$ or $\cos x = 0$

$\sin^2 x = \frac{2}{3}$

$\sin x = \pm\sqrt{\frac{2}{3}}$

Basic angle, $\alpha = \sin^{-1}\sqrt{\frac{2}{3}}$ or Basic angle, $\alpha = \frac{\pi}{2}$

$= 0.95532$

New range: $-3.2 < 0.6y - 1.4 < 1$

Replaced x by $(0.6y - 1.4)$,

$0.6y - 1.4 = 0.95532,$ or $0.6y - 1.4 = -\frac{\pi}{2}$

$-0.95532, -(\pi - 0.95532)$

$\therefore y = -1.31, 0.741, 3.93$ (3sf) $\quad \therefore y = -0.284$ (3sf)

Answer: $-1.31, 0.741, 3.93$

7(a) $\frac{dy}{dx} = \frac{(\cos x)(\cos x) - (1+\sin x)(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin x + \sin^2 x}{\cos^2 x}$

$= \frac{1+\sin x}{\cos^2 x}$

(b) From (a),

$\int_0^{\frac{\pi}{3}} \frac{1+\sin x}{\cos^2 x}\,dx = \left[\frac{1+\sin x}{\cos x}\right]_0^{\frac{\pi}{3}}$

$= \frac{1+\sin(\frac{\pi}{3})}{\cos(\frac{\pi}{3})} - \frac{1+\sin 0}{\cos 0}$

$= \frac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}} - 1$

$= 1 + \sqrt{3}$

Suggested Solutions for 2024 NCHS Prelim Exam Add Math Paper 1 (4049/1)

$\int_0^{\frac{\pi}{3}} \frac{3 + 3\sin x - 10\cos^3 x}{5\cos^2 x}\,dx$

$= \int_0^{\frac{\pi}{3}} \frac{3+3\sin x}{5\cos^2 x} - 2\cos x\,dx$

$= \frac{3}{5}\int_0^{\frac{\pi}{3}} \frac{1+\sin x}{\cos^2 x}\,dx - \int_0^{\frac{\pi}{3}} 2\cos x\,dx$

$= \frac{3}{5}(1+\sqrt{3}) - [2\sin x]_0^{\frac{\pi}{3}}$

$= \frac{3}{5} + \frac{3}{5}\sqrt{3} - (\sqrt{3} - 0)$

$= \frac{3}{5} - \frac{2}{5}\sqrt{3}$

$p = \frac{3}{5}, q = -\frac{2}{5}$

8(a) Sub $t = 0$, $h = 5$ m

(b) $h = -4.9\left(t^2 - \frac{80}{49}t\right) + 5$

$= -4.9\left[\left(t - \frac{40}{49}\right)^2 - \left(-\frac{40}{49}\right)^2\right] + 5$

$= -4.9\left(t - \frac{40}{49}\right)^2 + \frac{160}{49} + 5$

$= 8\frac{13}{49} - 4.9\left(t - \frac{40}{49}\right)^2$

(c) Greatest height = 8 13/[illegible] m; $t = \frac{40}{49}$ s

(d) Sub $h = 0$.

$8\frac{13}{49} - 4.9\left(t - \frac{40}{49}\right)^2 = 0$

$\left(t - \frac{40}{49}\right)^2 = 1.68680$

$t = 2.12$ s or -0.482 (rejected)

9 $5^p = 5^2 \times 5^{3q}$

$p = 2 + 3q$ ………(1)

$\log_3 7 - \log_3(11q - 2p) = 1$

$\log_3 \frac{7}{11q - 2p} = 1$

	$\frac{7}{11q-2p}=3^1$ $7=33q-6p$ ....(2) Sub (1) into (2), $7=33q-6(2+3q)$ $q=\frac{19}{15}$ Sub $q=\frac{19}{15}$ into (1), $p=5\frac{4}{5}$
10 (a)	$\frac{dy}{dx}=x^3\left(\frac{1}{x}\right)+(\ln x)(3x^2)$ $=3x^2\ln x+x^2$ $=x^2(3\ln x+1)$ At stationary point, $\frac{dy}{dx}=0$ $x^2(3\ln x+1)=0$ $x^2=0$ or $3\ln x+1=0$ $x=0$ (rejected) $\ln x=-\frac{1}{3}$ $x=e^{-\frac{1}{3}}$ Sub $x=e^{-\frac{1}{3}}$ into $y=x^3\ ln\ x$, $y=e^{-1}\times\ln e^{-\frac{1}{3}}$ $=-\frac{1}{3e}$ Coordinates $=\left(e^{-\frac{1}{3}},-\frac{1}{3e}\right)$
(b)	$\frac{d^2y}{dx^2}=2x(3\ln x+1)+x^2\ ($ $=6x\ ln\ x+5x$ Sub $x=e^{-\frac{1}{3}}$, $\frac{d^2y}{dx^2}=6e^{-\frac{1}{3}}\ln e^{-\frac{1}{3}}+5e^{-\frac{1}{3}}=2.15>0$ Hence, $\left(e^{-\frac{1}{3}},-\frac{1}{3e}\right)$ is a minimum point.
(c)	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $0.3x^2=\frac{dy}{dx}\times 0.06$ $\frac{dy}{dx}=5x^2$ $3x^2\ln x+x^2=5x^2$

	$x^2(3\ln x-4)=0$ $x^2=0$ or $3\ln x-4=0$ $x=0$ (rejected) or $x=e^{\frac{4}{3}}$ Sub $x=e^{\frac{4}{3}}$ into y, $y=e^4\ln e^{\frac{4}{3}}$ $=\frac{4}{3}e^4$ $M=(e^{\frac{4}{3}},\frac{4}{3}e^4)$
11 (a)	Centre $=(\frac{k}{-2},\frac{-6}{-2})$ $(-2,m)=(\frac{k}{-2},3)$ By comparing, $k=4$ $m=3$ Radius $=\sqrt{(-2)^2+(3)^2-(-h)}$ $7=\sqrt{13+h}$ $h=36$ Alternative method: $(x+2)^2+(y-m)^2=7^2$ $x^2+y^2+4x-2my=49-4-m^2$ Compare with $x^2+y^2+kx-6y=h$ $\therefore k=4$ $-2m=-6\Rightarrow m=3$ $h=49-4-m^2$ $=49-4-3^2$ $=36$
(bi)	Sub $x=3$, $4y+9=5$ $y=-1$ $P=(3,-1)$ Centre = midpoint of PQ $=\left(\frac{3+(-5)}{2},\frac{-1+5}{2}\right)$ $=(-1,2)$ Radius $=\frac{1}{2}\sqrt{(3+5)^2+(-1-5)^2}$ $=5$ Equation of C_2: $(x+1)^2+(y-2)^2=25$

(bii) Distance of S from centre of C_1 (-2, 3)

$= \sqrt{(4+2)^2 + (5-3)^2}$

$= 6.32 < 7$ (radius of C_1)

Distance of S from centre of C_2 (-1, 2)

$= \sqrt{(4+1)^2 + (5-2)^2}$

$= 5.83 > 5$ (radius of C_2)

Hence S only lies inside C_1 but not C_2.

12 (a) Let D be $(x, 0)$

Midpoint of AC = Midpoint of BD

$\left(\frac{x+2}{2}, \frac{0+7}{2}\right) = \left(\frac{-1+4}{2}, \frac{4+3}{2}\right)$

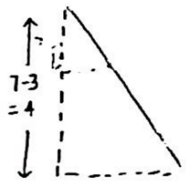
$x = 1 \implies D = (1, 0)$

By similar triangles,

$\frac{x-2}{2} = \frac{2}{5}$

$5x - 10 = 4$

$x = 2\frac{4}{5}$



$\frac{7-y}{4} = \frac{2}{5}$

$35 - 5y = 8$

$y = 5\frac{2}{5}$

$E = (2\frac{4}{5}, 5\frac{2}{5})$

Equation of DC:

$m_{DC} = m_{AB} = \frac{7-4}{2-(-1)} = 1$

Sub (4, 3), m = 1,

$y - 3 = 1(x - 4)$

$y = x - 1$..........(1)

Sub $x = 2\frac{4}{5}$ into (1),

$y = 1\frac{4}{5}$

$F = (2\frac{4}{5}, 1\frac{4}{5})$

(b) $G = (-1, \frac{2}{5})$

$m_{AE} = \frac{5.4-4}{2.8+1} = \frac{7}{19}$

$m_{GF} = \frac{1.8-\frac{2}{5}}{2.8+1} = \frac{7}{19}$

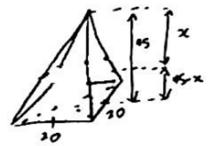
Hence $m_{AB} = m_{GE}$ and $m_{AG} = m_{EF}$ (Vertical lines)

Since there are 2 pairs of opposite parallel lines, $AEFG$ is a **parallelogram**.

13 (a) By similar triangles,

$\frac{s}{20} = \frac{x}{45}$

$s = \frac{4}{9}x$



Volume of sand, V

= Volume of big pyramid − volume of small pyramid

$= \frac{1}{3}(20)^2(45) - \frac{1}{3}\left(\frac{4}{9}x\right)^2(x)$

$= 6000 - \frac{16}{243}x^3$ (Shown)

(b) $\frac{dV}{dx} = -\frac{16}{81}x^2$

$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$

$b\,x^2 = -\frac{16}{81}x^2 \times \frac{dx}{dt}$

$\frac{dx}{dt} = -\frac{81b}{16}$

$x = \int -\frac{81b}{16}\,dt$

$= -\frac{81b}{16}t + c$

Sub $t = 0, x = 45$

$c = 45$

Hence $x = -\frac{81b}{16}t + 45$

Given $t = 24$, $45 - x = 36 \Rightarrow x = 9$

Sub $t = 24, x = 9$,

$9 = -\frac{81b}{16} \times 24 + 45$

$b = \frac{8}{27}$

$\therefore \frac{dx}{dt} = -\frac{81b}{16}$

$= -\frac{81}{16} \times \frac{8}{27}$

$= -1.5$ cm/s